Examples 2 in The Cosine Rule

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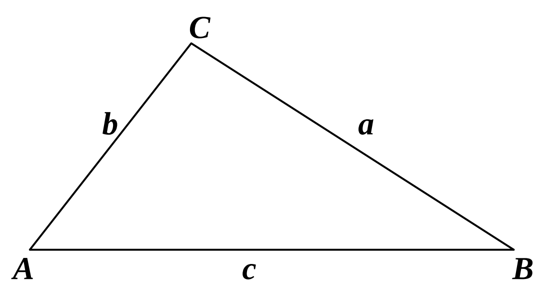
Doing all these examples, refer to the triangle ***ABC*** below.

Fig. 0



0. Assuming $\boldsymbol{a = x^2 + x + 1}$, $\boldsymbol{b = x^2 - 1}$, and $\boldsymbol{c = 2x + 1}$ for $\boldsymbol{x}$ real, determine the value of $\boldsymbol{x}$ so that $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ are the three sides of the triangle ***ABC***.

1. Assuming $\boldsymbol{a = x^2 + x + 1}$, $\boldsymbol{b = x^2 - 1}$, and $\boldsymbol{c = 2x + 1}$ for $\boldsymbol{x}$ real, and $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ are the three sides of the triangle ***ABC***, find the biggest of the three angles in the triangle ***ABC***.

2. Find the kind of the triangle ***ABC*** that satisfies an equation $\boldsymbol{\sin A = 2(\cos B)(\sin C)}$.

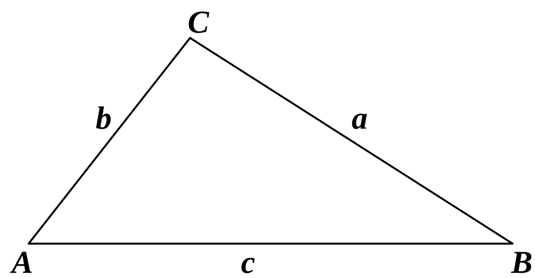
3. Find the kind of the triangle ***ABC*** that satisfies an equation $\boldsymbol{a \cos A = b \cos B}$.

4. Find the kind of the triangle ***ABC*** that satisfies $\boldsymbol{(b - c) \cos^2 A = b \cos^2 B - c \cos^2 C}$.

Suggestions or Solutions
To the Problem in the Example 0

Assuming $\boldsymbol{a} = x^2 + x + 1$, $\boldsymbol{b} = x^2 - 1$, and $c = 2x + 1$ for x real, determine the value of x so that $\boldsymbol{a}$, $\boldsymbol{b}$, and c are the three sides of the triangle *ABC* below.

Fig. 0.0



Forming a triangle with three line segments, we cannot just use any line segments.
What line segments then, can we use forming a triangle?
In other words, what is the condition on the line segments $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ if they can form a triangle?

The three line segments have to meet the triangle inequality, which says
$\boldsymbol{a} + \boldsymbol{b} > \boldsymbol{c}$, $\boldsymbol{b} + \boldsymbol{c} > \boldsymbol{a}$, or $\boldsymbol{c} + \boldsymbol{a} > \boldsymbol{b}$. Which one of the three though, we want to apply?

Assuming $\boldsymbol{a}$ is the largest of all the three sides, we get $\boldsymbol{b} + \boldsymbol{c} > \boldsymbol{a}$, which is the one to be applied. That is to say that if it turns out that $\boldsymbol{a}$ is the largest, we want to apply the inequality above. Is the equality all then, that we need to apply?

We need to have $\boldsymbol{a} > \mathbf{0}$, $\boldsymbol{b} > \mathbf{0}$, and $\boldsymbol{c} > \mathbf{0}$, of course, which is natural, since sides are positive. Thus, the set of three inequalities is the basic condition that the three line segments $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ have to meet. Let's now, begin with the basic condition.

What then do we need to begin with?

To begin with, we have $\boldsymbol{a} = \boldsymbol{x}^2 + \boldsymbol{x} + \mathbf{1}$, $\boldsymbol{b} = \boldsymbol{x}^2 - \mathbf{1}$, and $\boldsymbol{c} = \mathbf{2}\boldsymbol{x} + \mathbf{1}$ for $\boldsymbol{x}$ real.

So first, beginning with the basic condition stated above, we get these:

$\boldsymbol{a} = \boldsymbol{x}^2 + \boldsymbol{x} + \mathbf{1} = (\boldsymbol{x} + \mathbf{1/2})^2 + \mathbf{3/4} > \mathbf{0}$ for all $\boldsymbol{x}$.

$\boldsymbol{b} = \boldsymbol{x}^2 - \mathbf{1} = (\boldsymbol{x} - \mathbf{1})(\boldsymbol{x} + \mathbf{1}) > \mathbf{0} \Rightarrow \boldsymbol{x} > \mathbf{1}$ or $\boldsymbol{x} < \mathbf{-1}$.

$\boldsymbol{c} = \mathbf{2}\boldsymbol{x} + \mathbf{1} > \mathbf{0} \Rightarrow \boldsymbol{x} > \mathbf{-1/2}$.

Thus, $\boldsymbol{x}$ has to meet, at the same time, all the three cases below.
One is that $\boldsymbol{x}$ is real. Another is that $\boldsymbol{x} > \mathbf{1}$ or $\boldsymbol{x} < \mathbf{-1}$. And the other is that $\boldsymbol{x} > \mathbf{-1/2}$.

Then, getting the extent satisfying all three at the same time, we get $\boldsymbol{x} > \mathbf{1}$.
What then is the next?

We have to meet the triangle inequality. So first, finding the largest side, we get

$\boldsymbol{a} - \boldsymbol{b} = \boldsymbol{x}^2 + \boldsymbol{x} + \mathbf{1} - (\boldsymbol{x}^2 - \mathbf{1}) = \boldsymbol{x} + \mathbf{2} > \mathbf{0}$ since $\boldsymbol{x} > \mathbf{1}$, and thus, $\boldsymbol{a} > \boldsymbol{b}$.

$\boldsymbol{a} - \boldsymbol{c} = \boldsymbol{x}^2 + \boldsymbol{x} + \mathbf{1} - (\mathbf{2}\boldsymbol{x} + \mathbf{1}) = \boldsymbol{x}^2 - \boldsymbol{x} = \boldsymbol{x}(\boldsymbol{x} - \mathbf{1}) > \mathbf{0}$ since $\boldsymbol{x} > \mathbf{1}$, and thus, $\boldsymbol{a} > \boldsymbol{c}$.

So $\boldsymbol{a}$ is the largest. Thus, we want to apply this inequality $\boldsymbol{b} + \boldsymbol{c} > \boldsymbol{a}$.
And applying the triangle inequality, we can get first, $\boldsymbol{b} + \boldsymbol{c} > \boldsymbol{a} \Rightarrow \boldsymbol{b} + \boldsymbol{c} - \boldsymbol{a} > \mathbf{0}$.

And we have $\boldsymbol{a} = \boldsymbol{x}^2 + \boldsymbol{x} + \mathbf{1}$, $\boldsymbol{b} = \boldsymbol{x}^2 - \mathbf{1}$, and $\boldsymbol{c} = \mathbf{2}\boldsymbol{x} + \mathbf{1}$.

So we get $\boldsymbol{b} + \boldsymbol{c} - \boldsymbol{a} = (\boldsymbol{x}^2 - \mathbf{1}) + (\mathbf{2}\boldsymbol{x} + \mathbf{1}) - (\boldsymbol{x}^2 + \boldsymbol{x} + \mathbf{1}) = \boldsymbol{x} - \mathbf{1} > \mathbf{0} \Rightarrow \boldsymbol{x} > \mathbf{1}$.

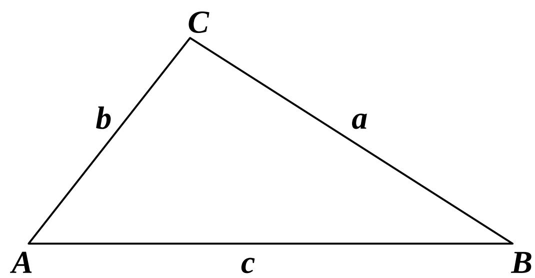
Also, we get $\boldsymbol{x} > \mathbf{1}$ from the basic condition, too.

Thus, putting threads together, we get $\boldsymbol{x} > \mathbf{1}$.

Suggestions or Solutions
To the Problem in the Example 1

Assuming $a = x^2 + x + 1$, $b = x^2 – 1$, and $c = 2x + 1$ for $x > 1$, and a, b, and c are the three sides of the triangle ABC below, find the biggest of the three angles in the triangle ABC.

Fig. 1.0



To begin with, we get $a – b = x^2 + x + 1 – (x^2 – 1) = x + 2 > 0$ for $x > 1$, and also, $a – c = x^2 + x + 1 – (2x + 1) = x^2 – x = x(x – 1) > 0$ for $x > 1$.

So the angle A is the biggest of the three angles,
since it faces the largest side, which is a. Next, we have $a^2 = b^2 + c^2 – 2bc \cos A$.

So we get $\cos A = \dfrac{b^2+c^2-a^2}{2bc} \Rightarrow \cos A = \dfrac{(x^2-1)^2+(2x+1)^2-(x^2+x+1)^2}{2(x^2-1)(2x+1)}$.

Meanwhile, we can have $(x^2 – 1)^2 + (2x + 1)^2 – (x^2 + x + 1)^2$

$= x^4 – 2x^2 + 1 + 4x^2 + 4x + 1 – (x^4 + x^2 + 1 + 2x^3 + 2x + 2x^2)$

$= 2x + 1 – (x^2 + 2x^3) = -2x^3 – x^2 + 2x + 1$, and also, we can get

$2(x^2 – 1)(2x + 1) = 2(2x^3 + x^2 – 2x – 1)$.

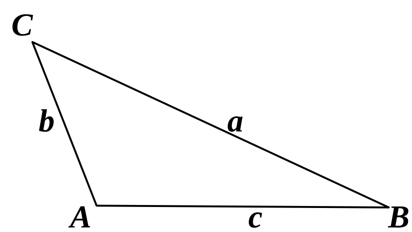
So we get $\cos A = \dfrac{-2x^3-x^2+2x+1}{2(2x^3+x^2-2x-1)} = \dfrac{-(2x^3+x^2-2x-1)}{2(2x^3+x^2-2x-1)} = -\dfrac{1}{2}$.

And we have $\cos \pi/3 = 1/2$. So we get $-\cos \pi/3 = -1/2$.
Also, we have $\cos(\pi – \theta) = -\cos \theta$. Thus, we get $A = \pi – \pi/3 = 2\pi/3$.

If not quite sure of the idea behind the processes above, follow the steps below.

To begin with, we have a fact that in the triangle, the longest side faces the largest angle of the three angles.
For instance, assuming $\boldsymbol{A}$ is the largest angle, we can say that the triangle is as follows.

Fig. 1.1



And in fact, we get

$\boldsymbol{a - b = x^2 + x + 1 - (x^2 - 1) = x + 2 > 0}$ for $\boldsymbol{x > 1}$, and also,

$\boldsymbol{a - c = x^2 + x + 1 - (2x + 1) = x^2 - x = x(x - 1) > 0}$ for $\boldsymbol{x > 1}$.

So we can see $\boldsymbol{A}$ is the biggest of the three angles, and faces the largest side, which is $\boldsymbol{a}$.
And in fact, the larger the angle is, the larger the facing side is.
How can we get the angle $\boldsymbol{A}$ though?

We have the cosine rule, where $\boldsymbol{a^2 = b^2 + c^2 - 2bc \cos A}$.
And we have this, too: $\boldsymbol{a = x^2 + x + 1}$, $\boldsymbol{b = x^2 - 1}$, and $\boldsymbol{c = 2x + 1}$ for $\boldsymbol{x > 1}$.

So applying the cosine rule, we get this first: $\boldsymbol{\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}}$, and thus, can set

$$\boldsymbol{\cos A = \frac{(x^2-1)^2 + (2x+1)^2 - (x^2+x+1)^2}{2(x^2-1)(2x+1)}}.$$

Meanwhile, we can have $\boldsymbol{(x^2 - 1)^2 + (2x + 1)^2 - (x^2 + x + 1)^2}$

$\boldsymbol{= x^4 - 2x^2 + 1 + 4x^2 + 4x + 1 - (x^4 + x^2 + 1 + 2x^3 + 2x + 2x^2)}$

$\boldsymbol{= 2x + 1 - (x^2 + 2x^3) = -2x^3 - x^2 + 2x + 1}$.

Also, we can get $\mathbf{2(x^2 - 1)(2x + 1) = 2(2x^3 + x^2 - 2x - 1)}$.

So we get $\mathbf{\cos A} = \dfrac{-2x^3 - x^2 + 2x + 1}{2(2x^3 + x^2 - 2x - 1)} = \dfrac{-(2x^3 + x^2 - 2x - 1)}{2(2x^3 + x^2 - 2x - 1)} = -\dfrac{1}{2}$.

What then is the angle $\boldsymbol{A}$?

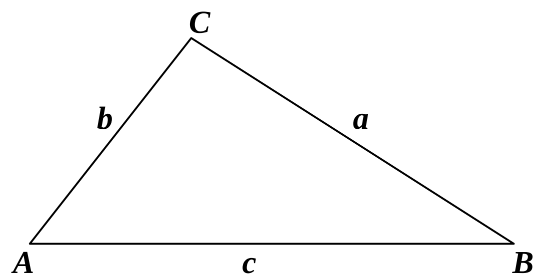
We have a trig identity where $\mathbf{\cos(\pi - \theta) = -\cos\theta}$.
And we have $\mathbf{\cos \pi/3 = 1/2}$, too.
So we get $\mathbf{-\cos \pi/3 = -1/2}$.

Thus, we get $\boldsymbol{A} \mathbf{= \pi - \pi/3 = 2\pi/3}$.

Suggestions or Solutions
To the Problem in the Example 2

Find the kind of the triangle *ABC* that satisfies an equation: sin *A* = 2(cos *B*)(sin *C*).

Fig. 2.0



We have $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$, and $b^2 = c^2 + a^2 - 2ca\cos B$.

So first, we can get $\sin A = a/(2R)$, $\cos B = \frac{c^2 + a^2 - b^2}{2ca}$, and $\sin C = (c/2R)$.

So next, putting the above into the equation where $\sin A = 2(\cos B)(\sin C)$, we get

$$\frac{a}{2R} = \frac{2(c^2 + a^2 - b^2)}{2ca} \cdot \frac{c}{2R} \Rightarrow a = \frac{(c^2 + a^2 - b^2)}{a}.$$

Thus, we get $a^2 = c^2 + a^2 - b^2 \Rightarrow c^2 = b^2 \Rightarrow c = b$.

Assuming now, $a = b = c$, we get $\cos B = \frac{c^2 + a^2 - b^2}{2ca} = \frac{a^2 + a^2 - a^2}{2aa} = \frac{1}{2}$.

So we get $B = \pi/3$. And next, we have $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$.

Setting thus, $a = b = c$, we get $\sin A = \sin B = \sin C$. So we get $A = C = \pi/3$, too.

Thus, it is the case where $a = b = c$. So the triangle ***ABC*** is a regular triangle.

If not quite sure of the idea behind the processes above, follow the steps below.

How do we know if a triangle is in a particular kind?

We could see it finding the relation among the sides. How then, can we find the relation?

We are given an equation made of **sin** $\boldsymbol{A}$, **cos** $\boldsymbol{B}$, and **sin** $\boldsymbol{C}$. So what?

We can put the equation in terms of the three sides, $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$.

That is to say that we can get a connective expression between the three sides.
And the expression can show the relation. How then can we get the expression?

The sine rule has **sin** $\boldsymbol{A}$ and **sin** $\boldsymbol{C}$, together with **sin** $\boldsymbol{B}$.

And the cosine rule has **cos** $\boldsymbol{B}$, together with the three sides, $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$.

The sine rule is $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2R$.

And the cosine rule is $b^2 = c^2 + a^2 - 2ca \cos B$.
So getting the expression we need, we can use the sine rule and the cosine rule. How?

Using the sine rule first, we can get $\sin A = \dfrac{a}{2R}$, and $\sin C = \dfrac{c}{2R}$.

And next, using the cosine rule, we can get $\cos B = \dfrac{c^2 + a^2 - b^2}{2ca}$.

So next, putting the expressions above into the equation where $\sin A = 2(\cos B)(\sin C)$,

we get $\dfrac{a}{2R} = \dfrac{2(c^2 + a^2 - b^2)}{2ca} \cdot \dfrac{c}{2R} \Rightarrow a = \dfrac{(c^2 + a^2 - b^2)}{a}$.

Thus, we get $a^2 = c^2 + a^2 - b^2 \Rightarrow c^2 = b^2 \Rightarrow c = b$. What triangle then is it?

It is a triangle where two sides are equal, and thus, is an isosceles triangle.
Can it not be though, a regular triangle?

Assuming now, $\boldsymbol{a = b = c}$, we get $\mathbf{cos}\ \boldsymbol{B} = \dfrac{c^2 + a^2 - b^2}{2ca} = \dfrac{a^2 + a^2 - a^2}{2aa} = \dfrac{1}{2}$.

So we get $\boldsymbol{B = \pi/3}$, which is 60°, which indicates it can be a regular triangle.

And next, we have $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2R$.

Setting thus, $\boldsymbol{a = b = c}$, we get $\mathbf{sin}\ \boldsymbol{A} = \mathbf{sin}\ \boldsymbol{B} = \mathbf{sin}\ \boldsymbol{C}$.

We know $\boldsymbol{B = \pi/3}$.   So we get $\boldsymbol{A = C = \pi/3}$, too.

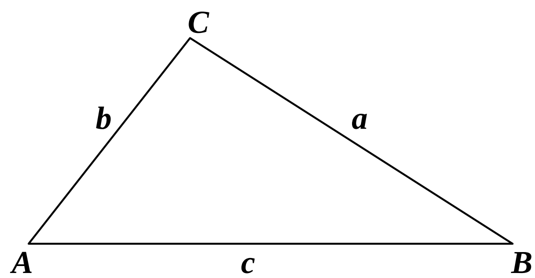
Thus, it is the case where $\boldsymbol{a = b = c}$.   So the triangle $\boldsymbol{ABC}$ is a regular triangle.

Note that a regular triangle is an isosceles triangle because two sides are equal in length in a regular triangle. That is, all regular triangles can be said to be isosceles. It is the case however, an isosceles triangle may not be a regular triangle.

Suggestions or Solutions
To the Problem in the Example 3

Find the kind of the triangle ABC that satisfies an equation $a \cos A = b \cos B$.

Fig. 3.0



We have the cosine rule, where $a^2 = b^2 + c^2 - 2bc \cos A$, and $b^2 = c^2 + a^2 - 2ca \cos B$.

So we get $a \cos A = \dfrac{a(b^2 + c^2 - a^2)}{2bc}$, and $b \cos B = \dfrac{b(c^2 + a^2 - b^2)}{2ca}$.

And we have $a \cos A = b \cos B$. Thus, we get

$$\frac{a(b^2 + c^2 - a^2)}{2bc} = \frac{b(c^2 + a^2 - b^2)}{2ca} \Rightarrow \frac{a(b^2 + c^2 - a^2)}{b} = \frac{b(c^2 + a^2 - b^2)}{a}.$$

So multiplying both sides by ab, we get

$$a^2(b^2 + c^2 - a^2) = b^2(c^2 + a^2 - b^2)$$

$$\Rightarrow a^2(b^2 + c^2 - a^2 - 2b^2 + 2b^2) = b^2(c^2 + a^2 - b^2 - 2a^2 + 2a^2)$$

$$\Rightarrow a^2(c^2 - a^2 - b^2) + 2a^2b^2 = b^2(c^2 - a^2 - b^2) + 2a^2b^2$$

$$\Rightarrow a^2(c^2 - a^2 - b^2) = b^2(c^2 - a^2 - b^2)$$

$$\Rightarrow (a^2 - b^2)(c^2 - a^2 - b^2) = 0 \Rightarrow a^2 = b^2 \text{ or } c^2 = a^2 + b^2.$$

Therefore, it can be any of the three cases below.

- It is an isosceles triangle with $\boldsymbol{a = b}$.
- It is a right triangle with $\boldsymbol{C = \pi/2}$.
- It is an isosceles right triangle with $\boldsymbol{C = \pi/2}$.

And of course, we can do the algebra the way below, too.

$$a^2(b^2 + c^2 - a^2) = b^2(c^2 + a^2 - b^2)$$

$$\Rightarrow a^2b^2 + a^2c^2 - a^4 = b^2c^2 + b^2a^2 - b^4 \Rightarrow a^2c^2 - a^4 = b^2c^2 - b^4$$

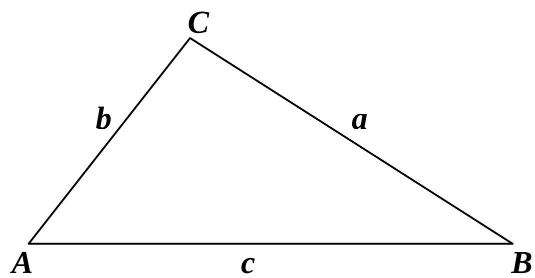
$$\Rightarrow a^2c^2 - a^4 - b^2c^2 + b^4 = c^2(a^2 - b^2) - (a^4 - b^4) = c^2(a^2 - b^2) - (a^2 - b^2)(a^2 + b^2)$$

$$= (a^2 - b^2)(c^2 - a^2 - b^2) = 0 \Rightarrow a^2 = b^2 \text{ or } c^2 = a^2 + b^2.$$

Suggestions or Solutions
To the Problem in the Example 4

Find the kind of the triangle *ABC* that satisfies $(b - c)\cos^2 A = b\cos^2 B - c\cos^2 C$**.**

Fig. 4.0



The equation given is put in terms of the sides and the cosines of the three angles.
So it looks like we want to try the cosine rule.

The algebra gets easier though, applying the sine rule first.
How then can we apply the sine rule?

We have a trig-identity where $\sin^2\theta + \cos^2\theta = 1$.

So applying first, the identity above, we can get $(b - c)\cos^2 A = b\cos^2 B - c\cos^2 C$

$\Rightarrow (b - c)(1 - \sin^2 A) = b(1 - \sin^2 B) - c\,(1 - \sin^2 C)$.

Meanwhile, we can have $(b - c)(1 - \sin^2 A) = (b - c) - (b - c)\sin^2 A$, and

$b(1 - \sin^2 B) - c\,(1 - \sin^2 C) = (b - c) - b\sin^2 B + c\sin^2 C$.

So we get $(b - c) - (b - c)\sin^2 A = (b - c) - b\sin^2 B + c\sin^2 C$

$\Rightarrow -(b - c)\sin^2 A = -b\sin^2 B + c\sin^2 C$.

That is, we get $(b - c)\sin^2 A = b\sin^2 B - c\sin^2 C$….(1)   What then is the next?

We have the sine rule, where $\dfrac{\boldsymbol{a}}{\sin \boldsymbol{A}} = \dfrac{\boldsymbol{b}}{\sin \boldsymbol{B}} = \dfrac{\boldsymbol{c}}{\sin \boldsymbol{C}} = 2\boldsymbol{R}$.

That is, we have $\sin \boldsymbol{A} = \dfrac{\boldsymbol{a}}{2\boldsymbol{R}}$, $\sin \boldsymbol{B} = \dfrac{\boldsymbol{b}}{2\boldsymbol{R}}$, and $\sin \boldsymbol{C} = \dfrac{\boldsymbol{c}}{2\boldsymbol{R}}$.

So from (1) above, we get $\dfrac{(\boldsymbol{b}-\boldsymbol{c})\boldsymbol{a}^2}{4\boldsymbol{R}^2} = \dfrac{\boldsymbol{bb}^2}{4\boldsymbol{R}^2} - \dfrac{\boldsymbol{cc}^2}{4\boldsymbol{R}^2}$. That is, we get $(\boldsymbol{b} - \boldsymbol{c})\boldsymbol{a}^2 = \boldsymbol{b}^3 - \boldsymbol{c}^3$.

And we have $\boldsymbol{b}^3 - \boldsymbol{c}^3 = (\boldsymbol{b} - \boldsymbol{c})(\boldsymbol{b}^2 + \boldsymbol{bc} + \boldsymbol{c}^2)$.

So we get $(\boldsymbol{b} - \boldsymbol{c})\boldsymbol{a}^2 = (\boldsymbol{b} - \boldsymbol{c})(\boldsymbol{b}^2 + \boldsymbol{bc} + \boldsymbol{c}^2) \Rightarrow (\boldsymbol{b} - \boldsymbol{c})(\boldsymbol{b}^2 + \boldsymbol{bc} + \boldsymbol{c}^2 - \boldsymbol{a}^2) = \mathbf{0}$.

Thus, we get $\boldsymbol{b} = \boldsymbol{c}$, which indicates an isosceles triangle, or we get $\boldsymbol{a}^2 = \boldsymbol{c}^2 + \boldsymbol{bc} + \boldsymbol{b}^2$, which does not indicate a triangle of any particular kind.

What then can we say?

We can notice though, that the expression has $\boldsymbol{a}^2 = \boldsymbol{c}^2 + \boldsymbol{b}^2$, which can be found in the cosine rule, where $\boldsymbol{a}^2 = \boldsymbol{b}^2 + \boldsymbol{c}^2 - 2\boldsymbol{bc} \cos \boldsymbol{A}$. So we may want to try the rule now. How can we try the rule though?

We can put the rule this way, too: $2\boldsymbol{bc} \cos \boldsymbol{A} = \boldsymbol{b}^2 + \boldsymbol{c}^2 - \boldsymbol{a}^2$.

And we can get $\boldsymbol{a}^2 = \boldsymbol{c}^2 + \boldsymbol{bc} + \boldsymbol{b}^2 \Rightarrow \boldsymbol{bc} = \boldsymbol{a}^2 - \boldsymbol{c}^2 - \boldsymbol{b}^2 \Rightarrow -\boldsymbol{bc} = \boldsymbol{b}^2 + \boldsymbol{c}^2 - \boldsymbol{a}^2$.

So applying the rule now to the above, we can get $-\boldsymbol{bc} = 2\boldsymbol{bc} \cos \boldsymbol{A} \Rightarrow \cos \boldsymbol{A} = -1/2$.

And we know $\cos \pi/3 = 1/2$. What then about $\cos \boldsymbol{A} = -1/2$?

We have a trig-identity where $\cos (\pi - \theta) = -\cos \theta$, and have $-\cos \pi/3 = -1/2$, too. So?

So we get **cos $(\pi - \pi/3) = -\cos \pi/3 = -1/2$**.

Thus, we get $\boldsymbol{A} = \pi - \pi/3 = 2\pi/3$, which is 120^o.

So it can be any of the three cases below.

- It is an isosceles triangle with $\boldsymbol{b} = \boldsymbol{c}$.

- It is a triangle with $\boldsymbol{A} = 2\pi/3$.

- It is an isosceles triangle with $\boldsymbol{A} = 2\pi/3$.