Examples 1 in Coordinate Trigonometry

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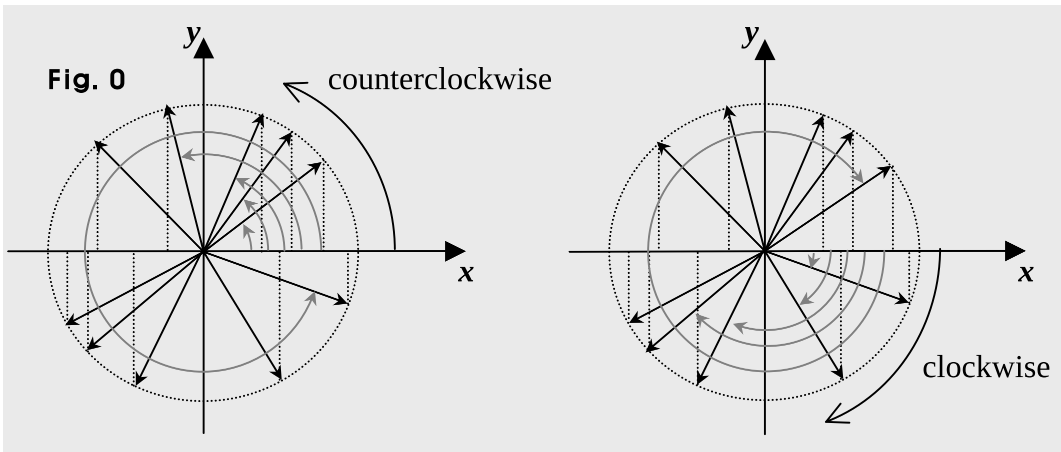


In coordinate trigonometry, the vector turning makes the angles.

If turning clockwise, it makes angles negative, and if counterclockwise, it makes angles positive. And of course, if no turning, the angle made is 0.

So an angle can be 0^o, positive, or negative.

In each right triangle, the vector is of length 1, and is the hypotenuse.

And (x, y) is the terminal point, so x is the adjacent, and y is the opposite.

Thus, assuming θ is a governing angle, we get $\sin\theta = y$, $\cos\theta = x$, and $\tan\theta = y/x$.

So if the vector is in the first quadrant, all the three trig-ratios > 0, since x and y both > 0.

In the second, only the sines > 0, since $x < 0$, and $y > 0$.

In the third, only the tangents > 0, since $x < 0$, and $y < 0$.

And in the fourth, only the cosines > 0, since $x > 0$, and $y < 0$.

0. Assuming $\mathbf{180° < \theta < 270^o}$, simplify the expression below:

$$\sqrt{\sin^2\theta} + \sqrt[3]{(\sin\theta + \cos\theta)^3} - \sqrt[4]{(\cos\theta + \tan\theta + 1)^4}$$

1. Find the value of each as follows: $\mathbf{\sin 420^o}$, $\mathbf{\cos 960^o}$, and $\mathbf{\tan(-1200^o)}$.

2. Find the value of $\mathbf{\sin\{n\pi/2 + (-1)^n(\pi/6)\}}$ where n is an integer.

Suggestions or Solutions
To the Problem in the Example 0

Assuming $180° < \theta < 270^o$, simplify the expression below.

$$\sqrt{\sin^2\theta} + \sqrt[3]{(\sin\theta + \cos\theta)^3} - \sqrt[4]{(\cos\theta + \tan\theta + 1)^4}$$

We know, for instance, $\sqrt{2^2} = 2$.

So assuming $P = \sqrt{\sin^2\theta} + \sqrt[3]{(\sin\theta + \cos\theta)^3} - \sqrt[4]{(\cos\theta + \tan\theta + 1)^4}$, can we just put P this way, too: $P = \sin\theta + (\sin\theta + \cos\theta) - (\cos\theta + \tan\theta + 1) = 2\sin\theta - \tan\theta - 1$?

We can do so if for instance, we have $0° < \theta < 90^o$.

If however, for instance, we have $200° < \theta < 220^o$, we cannot just remove the radical signs (the root signs) the way above.
That is to say that, for instance, we cannot just set $\sqrt{\sin^2\theta} = \sin\theta$. Why not?

That's because for instance, $\sqrt{x^2}$ is not just x but $|x|$.
It's because x can be negative, and we have $\sqrt{x^2} \geq 0$.
And if $180° < \theta < 270^o$, we get $-1 < \sin\theta < 0$.

So if $180° < \theta < 270^o$, we get $\sqrt{\sin^2\theta} = -\sin\theta$, since $0 < \sqrt{\sin^2\theta} < 1$.
And in fact, we want to set $\sqrt[n]{x^n} = |x|$ if n is an integer even. It's because if n is an even integer, we get $\sqrt[n]{x^n} \geq 0$, and x can be negative as well as 0 or positive. That is, if n is even, the sign of $\sqrt[n]{x^n}$ can be other than that of x. So for instance, $\sqrt[4]{(-2)^4} = 2$, and not -2.

If however, n is odd, we can just simply set $\sqrt[n]{x^n} = x$. It's because the sign of $\sqrt[n]{x^n}$ is the same as that of x. For instance, $\sqrt[3]{(-3)^3} = -3$. So we want to be careful when removing radical signs.

If not quite sure of radicals, refer to the book **POWERS AND LOGARITHMS**.

Now, if **180° < θ < 270°**, we get **-1 < sin θ < 0**, **-1 < cos θ < 0**, and **tan θ > 0**.
How though?

That's because if **180° < θ < 270°**, the vector turning in the ***x-y*** plane is in the third quadrant, the ***x***-coordinate of the terminal point is negative, and so is the ***y***-coordinate.
So what?

Assuming the vector is of length 1, and θ is the angle made by the vector, we can put the basic trig ratios the way as follows.
sin θ is the ***y***-coordinate of the terminal point, **cos θ** is the ***x***-coordinate of the terminal point, and **tan θ** is the ***y***-coordinate over the ***x***-coordinate.
In short, if **(*x*, *y*)** is the terminal point, we get **sin $\theta = y$**, **cos $\theta = x$**, and **tan $\theta = \frac{y}{x}$**.
And if the vector is in the third quadrant, so if **180° < θ < 270°**, we get ***x* < 0**, and ***y* < 0**.

And we know these:

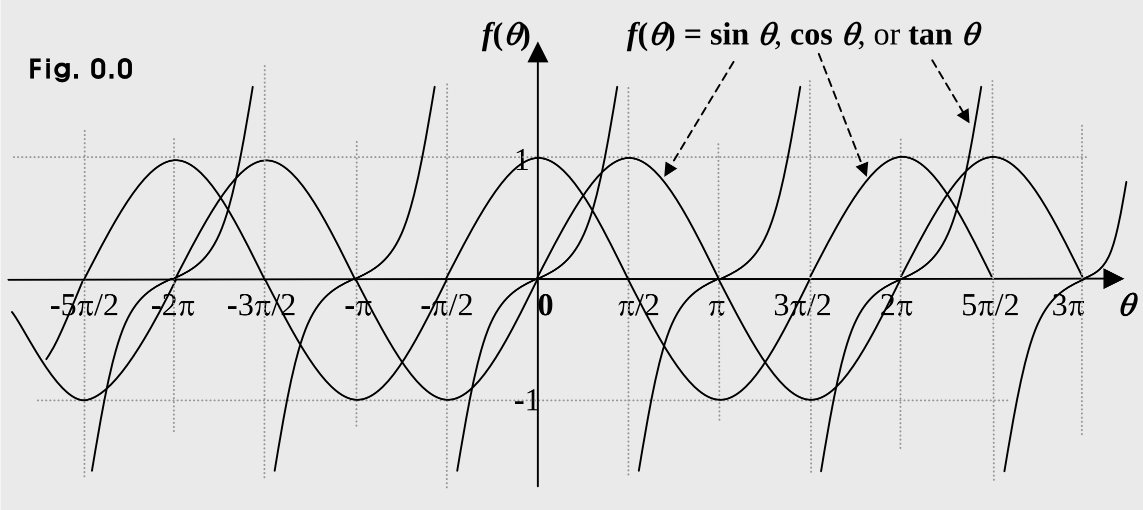
If **θ = 180°**, that is, π, we get: ***y* = 0**, and ***x* = -1**. So **sin θ = 0**, **cos θ = -1**, and **tan θ = 0** since **tan** θ = $\frac{y}{x}$ and ***y* = 0**..

If **θ = 270°**, that is, $\frac{3\pi}{2}$, we get ***y* = -1**, and ***x* = 0**. So **sin θ = -1**, **cos θ = 0**, and **tan θ** is not defined since **tan $\theta = \frac{y}{x}$** and ***x* = 0**.

And putting in graphs, the curves of **sin θ**, **cos θ**, and **tan θ**, we can see better how it is the case where if **π < $\theta < \frac{3\pi}{2}$**, we get **-1 < sin θ < 0**, **-1 < cos θ < 0**, and **tan θ > 0**.

And the curves will look like the ones below.

Fig. 0.0



Now, we have $\boldsymbol{P} = \sqrt{\sin^2\theta} + \sqrt[3]{(\sin\theta + \cos\theta)^3} - \sqrt[4]{(\cos\theta + \tan\theta + 1)^4}$.

And we have **$-1 < \sin\theta < 0$**, **$-1 < \cos\theta < 0$**, and **$\tan\theta > 0$**, since $\pi <$ **$\theta < 3\pi/2$**.

So we get **$\cos\theta + 1 > 0$** since **$-1 < \cos\theta < 0$**, and **$\cos\theta + \tan\theta + 1 > 0$**, since **$\tan\theta > 0$**.

Thus, we get **$P = -\sin\theta + (\sin\theta + \cos\theta) - (\cos\theta + \tan\theta + 1) = -\tan\theta - 1$**.

So we get **$P = -\tan\theta - 1$** if **$180° < \theta < 270^o$**.

Suggestions or Solutions

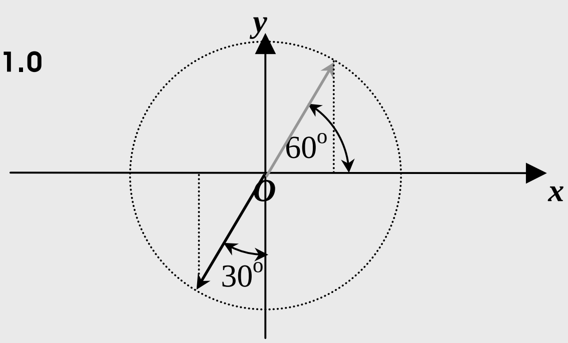
To the Problem in the Example 1

Find the values of sin 420^o, cos 960^o, and tan (-1200^o).

To begin with, we know 420 = 360 + 60.
So considering the vector turning in the ***x-y*** plane, we can say, in this case, the vector is in the first quadrant.
Thus, we get $\mathbf{\sin 420° = \sin (360 + 60)° = \sin 60° = \frac{\sqrt{3}}{2}}$.

Next, we know 960 = 900 + 60 = 10·90 + 60 = 5·180 + 60 = 2·360 + 1·180 + 60.
So the vector is in the third quadrant. And putting the vector in the ***x-y*** plane, we get

Fig. 1.0



So if the vector is of length 1, and (x, y) is the terminal point, we get $\mathbf{\cos (180 + 60)^o = x}$, and $\mathbf{\cos 60^o = -x}$.   How?

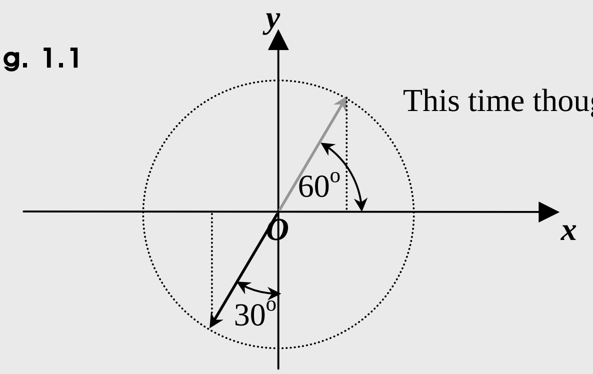
That's because the cosine is the ***x***-coordinate at the terminal point.

Thus, $\mathbf{\cos 960° = \cos (180 + 60)^o = -\cos 60^o = -1/2}$, since $\mathbf{\cos 60^o = 1/2}$.

And next, we know 1200 = 13·90 + 30 = 6·180 + 90 + 30 = 3·360 + 90 + 30.   So?

So -1200 = -3·360 – 90 – 30, and thus, the vector is in the third quadrant.

Fig. 1.1



This time though, the vector turns clockwise.

So assuming $\mathbf{tan\ (-90 - 30)^o} = \frac{y}{x}$, we get $\mathbf{tan\ 60^o} = \frac{y}{x}$, too. How?

That's because the tangent is the opposite over the adjacent, and thus, is the ***y***-coordinate over the ***x***-coordinate, at the terminal point, of course.

Thus, $\mathbf{tan\ (-1200^o) = tan\ (-90 - 30)^o = tan\ 60^o} = \sqrt{\mathbf{3}}$.

Suggestions or Solutions

To the Problem in the Example 2

Find the value of sin $\{n\pi/2 + (-1)^n \frac{\pi}{6}\}$ where n is an integer.

The angle depends on the value of n.

So let's first, put some values into n beginning with 0, and see how the angle changes.

- $n = 0 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = \pi/6$.

$n = 1 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = \pi/2 - \pi/6 = \pi/3$.

$n = 2 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 2\pi/2 + \pi/6 = \pi + \pi/6$.

$n = 3 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 3\pi/2 - \pi/6 = \pi + \pi/2 - \pi/6 = \pi + \pi/3$.

- $n = 4 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 4\pi/2 + \pi/6 = 2\pi + \pi/6$.

$n = 5 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 5\pi/2 - \pi/6 = 2\pi + \pi/2 - \pi/6 = 2\pi + \pi/3$.

$n = 6 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 6\pi/2 + \pi/6 = 2\pi + \pi + \pi/6$.

$n = 7 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 7\pi/2 - \pi/6 = 2\pi + 3\pi/2 - \pi/6 = 2\pi + \pi + \pi/3$.

- $n = 8 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 8\pi/2 + \pi/6 = 4\pi + \pi/6$.

$n = 9 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 9\pi/2 - \pi/6 = 4\pi + \pi/2 - \pi/6 = 4\pi + \pi/3$.

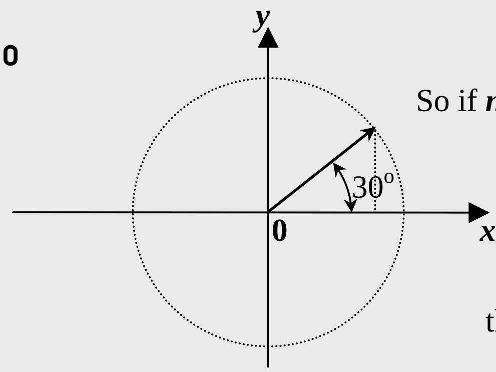
$n = 10 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 10\pi/2 + \pi/6 = 4\pi + \pi + \pi/6$.

$n = 11 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 11\pi/2 - \pi/6 = 4\pi + 3\pi/2 - \pi/6 = 4\pi + \pi + \pi/3$.

- $n = 12 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 12\pi/2 + \pi/6 = 6\pi + \pi/6$.

Now, taking a closer look at the sequence of angles above, we can notice that if $\boldsymbol{n = 4k}$ for $\boldsymbol{k}$ integer $\geq$ **0**, all the vectors get the same spot.

Fig. 2.0



So if $\boldsymbol{n = 4k}$ for $\boldsymbol{k}$ integer $\geq$ **0**, all the vectors get the same spot in the first quadrant.

Thus, if $\boldsymbol{n = 4k}$ for $\boldsymbol{k}$ integer $\geq$ **0**, the sine of $\boldsymbol{\{n\pi/2 + (-1)^n(\pi/6)\}}$ is the same, and is the same as $\boldsymbol{\sin \pi/6}$, which is 1/2.

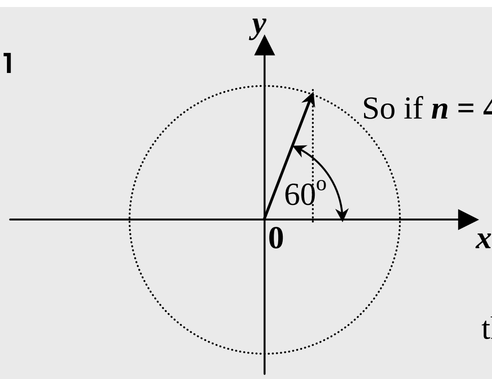
So if $\boldsymbol{n = 4k}$ for $\boldsymbol{k}$ integer $\geq$ **0**, $\boldsymbol{\sin \{n\pi/2 + (-1)^n(\pi/6)\} = \sin \pi/6 = 1/2}$.
Next, we can notice that if $\boldsymbol{n = 4k + 1}$ for $\boldsymbol{k}$ integer $\geq$ **0**, all the vectors get the same spot again.

$\boldsymbol{n = 1 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = \pi/2 - \pi/6 = \pi/3}$.

$\boldsymbol{n = 5 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 5\pi/2 - \pi/6 = 2\pi + \pi/2 - \pi/6 = 2\pi + \pi/3}$.

$\boldsymbol{n = 9 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 9\pi/2 - \pi/6 = 4\pi + \pi/2 - \pi/6 = 4\pi + \pi/3}$.

Fig. 2.1



So if $\boldsymbol{n = 4k + 1}$ for $\boldsymbol{k}$ integer $\geq$ **0**, all the vectors get the same spot in the first quadrant.

Thus, if $\boldsymbol{n = 4k + 1}$ for $\boldsymbol{k}$ integer $\geq$ **0**, the sine of $\boldsymbol{\{n\pi/2 + (-1)^n(\pi/6)\}}$ is the same, and is the same as $\boldsymbol{\sin (\pi/3)}$, which is $\frac{\sqrt{3}}{2}$.

So if $\boldsymbol{n = 4k + 1}$ for $\boldsymbol{k}$ integer $\geq$ **0**, $\boldsymbol{\sin \{n\pi/2 + (-1)^n(\pi/6)\} = \sin \pi/3} = \frac{\sqrt{3}}{2}$.

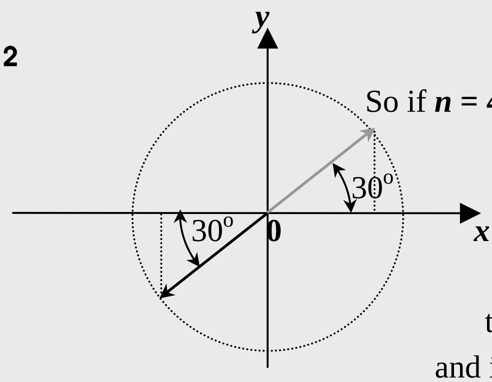
Next, we can notice that if $\boldsymbol{n = 4k + 2}$ for $\boldsymbol{k}$ integer $\geq$ **0**, all the vectors get the same spot again.

$\boldsymbol{n = 2 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 2\pi/2 + \pi/6 = \pi + \pi/6}$.

$\boldsymbol{n = 6 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 6\pi/2 + \pi/6 = 2\pi + \pi + \pi/6}$.

$\boldsymbol{n = 10 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 10\pi/2 + \pi/6 = 4\pi + \pi + \pi/6}$.

Fig. 2.2



So if $\boldsymbol{n = 4k + 2}$ for $\boldsymbol{k}$ integer $\geq$ **0**, all the vectors get the same spot in the third quadrant.

Thus, if $\boldsymbol{n = 4k + 2}$ for $\boldsymbol{k}$ integer $\geq$ **0**, the sine of $\boldsymbol{\{n\pi/2 + (-1)^n(\pi/6)\}}$ is the same, and is the same as $\boldsymbol{\sin(\pi + \pi/6)}$, which is -1/2. That's because $\boldsymbol{\sin(\pi + \pi/6) = -\sin \pi/6 = -1/2}$.

So if $\boldsymbol{n = 4k + 2}$ for $\boldsymbol{k}$ integer $\geq$ **0**, $\boldsymbol{\sin\{n\pi/2 + (-1)^n(\pi/6)\} = -\sin \pi/6 = -1/2}$.

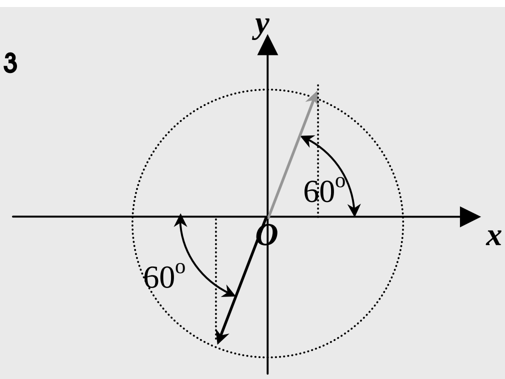
And next, we can notice that if $\boldsymbol{n = 4k + 3}$ for $\boldsymbol{k}$ integer $\geq$ **0**, all the vectors get the same spot again.

$\boldsymbol{n = 3 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 3\pi/2 - \pi/6 = \pi + \pi/2 - \pi/6 = \pi + \pi/3}$.

$\boldsymbol{n = 7 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 7\pi/2 - \pi/6 = 2\pi + 3\pi/2 - \pi/6 = 2\pi + \pi + \pi/3}$.

$\boldsymbol{n = 11 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 11\pi/2 - \pi/6 = 4\pi + 3\pi/2 - \pi/6 = 4\pi + \pi + \pi/3}$.

Fig. 2.3



So if $\boldsymbol{n = 4k + 3}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, all the vectors get the same spot in the third quadrant. Thus, if $\boldsymbol{n = 4k + 3}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, the sine of $\mathbf{\{n\pi/2 + (-1)^n(\pi/6)\}}$ is the same, and is the same as $\mathbf{\sin(\pi + \pi/3)}$, which is $-\frac{\sqrt{3}}{2}$.

That's because $\mathbf{\sin(\pi + \pi/3) = -\sin \pi/3} = -\frac{\sqrt{3}}{2}$.

So if $\boldsymbol{n = 4k + 3}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, $\mathbf{\sin\{n\pi/2 + (-1)^n(\pi/6)\} = -\sin \pi/3} = -\frac{\sqrt{3}}{2}$.

What if though, $\boldsymbol{k}$ is an integer negative?

Putting some negative values into $\boldsymbol{n}$ beginning with 0, we get a sequence below.

- $\boldsymbol{n = 0 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = \pi/6}$.

$\boldsymbol{n = -1 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -\pi/2 - \pi/6 = -2\pi/3}$.

$\boldsymbol{n = -2 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -2\pi/2 + \pi/6 = -\pi + \pi/6}$.

$\boldsymbol{n = -3 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -3\pi/2 - \pi/6 = -\pi - \pi/2 - \pi/6 = -\pi - 2\pi/3}$.

- $\boldsymbol{n = -4 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -4\pi/2 + \pi/6 = -2\pi + \pi/6}$.

$\boldsymbol{n = -5 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -5\pi/2 - \pi/6 = -2\pi - \pi/2 - \pi/6 = -2\pi - 2\pi/3}$.

$\boldsymbol{n = -6 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -6\pi/2 + \pi/6 = -2\pi - \pi + \pi/6}$.

$\boldsymbol{n = -7 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -7\pi/2 - \pi/6 = -2\pi - 3\pi/2 - \pi/6 = -2\pi - \pi - 2\pi/3}$.

- $\boldsymbol{n = -8 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -8\pi/2 + \pi/6 = -4\pi + \pi/6}$.

$\boldsymbol{n = -9 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -9\pi/2 - \pi/6 = -4\pi - \pi/2 - \pi/6 = -4\pi - 2\pi/3}$.

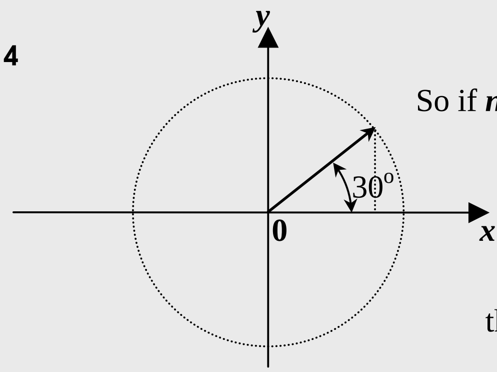
$\boldsymbol{n = -10 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -10\pi/2 + \pi/6 = -4\pi - \pi + \pi/6}$.

$\boldsymbol{n = -11 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -11\pi/2 - \pi/6 = -4\pi + 3\pi/2 - \pi/6 = -4\pi - \pi - 2\pi/3}$.

- $\boldsymbol{n = -12 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -12\pi/2 + \pi/6 = -6\pi + \pi/6}$.

Now, taking a closer look at the sequence of angles above, we can notice again, if $\boldsymbol{n} = \mathbf{4}\boldsymbol{k}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, all the vectors get the same spot again.

Fig. 2.4



So if $\boldsymbol{n} = \mathbf{4}\boldsymbol{k}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, all the vectors get the same spot in the first quadrant.

Thus, if $\boldsymbol{n} = \mathbf{4}\boldsymbol{k}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, the sine of $\{n\pi/2 + (-1)^n(\pi/6)\}$ is the same, and is the same as $\sin \pi/6$, which is 1/2.

So if $\boldsymbol{n} = \mathbf{4}\boldsymbol{k}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, $\sin \{n\pi/2 + (-1)^n(\pi/6)\} = 1/2$.

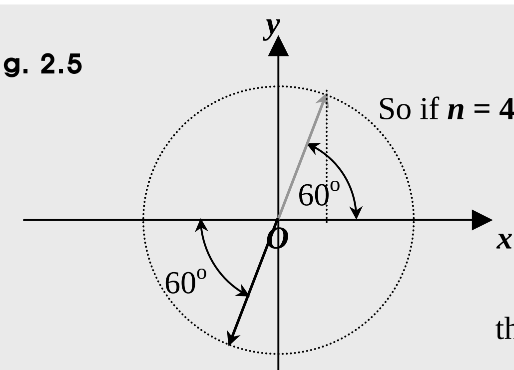
Next, we can notice that if $\boldsymbol{n} = \mathbf{4}\boldsymbol{k} - \mathbf{1}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, all the vectors get the same spot again.

$n = -1 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -\pi/2 - \pi/6 = -2\pi/3$.

$n = -5 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -5\pi/2 - \pi/6 = -2\pi - \pi/2 - \pi/6 = -2\pi - 2\pi/3$.

$n = -9 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -9\pi/2 - \pi/6 = -4\pi - \pi/2 - \pi/6 = -4\pi - 2\pi/3$.

Fig. 2.5



So if $\boldsymbol{n} = \mathbf{4}\boldsymbol{k} - \mathbf{1}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, all the vectors get the same spot in the third quadrant.

Thus, if $\boldsymbol{n} = \mathbf{4}\boldsymbol{k} - \mathbf{1}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, the sine of $\{n\pi/2 + (-1)^n(\pi/6)\}$ is the same, and is the same as $-\sin(\pi/3)$, which is $-\frac{\sqrt{3}}{2}$.

So if $\boldsymbol{n} = \mathbf{4}\boldsymbol{k} - \mathbf{1}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, $\sin \{n\pi/2 + (-1)^n(\pi/6)\} = -\sin \pi/3 = -\frac{\sqrt{3}}{2}$.

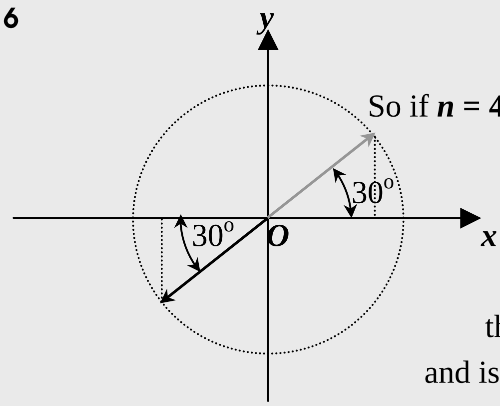
Next, we can notice that if $\boldsymbol{n = 4k - 2}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, all the vectors get the same spot again.

$\boldsymbol{n = -2 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -2\pi/2 + \pi/6 = -\pi + \pi/6}$.

$\boldsymbol{n = -6 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -6\pi/2 + \pi/6 = -2\pi - \pi + \pi/6}$.

$\boldsymbol{n = -10 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -10\pi/2 + \pi/6 = -4\pi - \pi + \pi/6}$.

Fig. 2.6



So if $\boldsymbol{n = 4k - 2}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, all the vectors get the same spot in the third quadrant.

Thus, if $\boldsymbol{n = 4k - 2}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, the sine of $\boldsymbol{\{n\pi/2 + (-1)^n(\pi/6)\}}$ is the same, and is the same as $\mathbf{sin}\ \boldsymbol{(-\pi + \pi/6)}$, which is -1/2. That's because $\mathbf{sin}\ \boldsymbol{(-\pi + \pi/6) = -}\mathbf{sin}\ \boldsymbol{\pi/6 = -1/2}$.

So if $\boldsymbol{n = 4k - 2}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, $\mathbf{sin}\ \boldsymbol{\{n\pi/2 + (-1)^n(\pi/6)\} = -}\mathbf{sin}\ \boldsymbol{\pi/6 = -1/2}$.

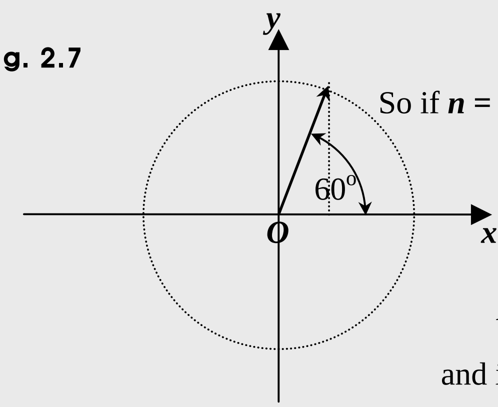
And next, we can notice that if $\boldsymbol{n = 4k - 3}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, all the vectors get the same spot again.

$\boldsymbol{n = -3 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -3\pi/2 - \pi/6 = -\pi - \pi/2 - \pi/6 = -\pi - 2\pi/3}$.

$\boldsymbol{n = -7 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -7\pi/2 - \pi/6 = -2\pi - 3\pi/2 - \pi/6 = -2\pi - \pi - 2\pi/3}$.

$\boldsymbol{n = -11 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -11\pi/2 - \pi/6 = -4\pi + 3\pi/2 - \pi/6 = -4\pi - \pi - 2\pi/3}$.

Fig. 2.7



So if $\boldsymbol{n = 4k - 3}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, all the vectors get the same spot in the third quadrant.

Thus, if $\boldsymbol{n = 4k - 3}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, the sine of $\boldsymbol{\{n\pi/2 + (-1)^n(\pi/6)\}}$ is the same, and is the same as **sin** $\boldsymbol{(-\pi - 2\pi/3)}$, which is $\frac{\sqrt{3}}{2}$.

That's because **sin** $\boldsymbol{(-\pi - 2\pi/3) =}$ **sin** $\boldsymbol{\pi/3 = \frac{\sqrt{3}}{2}}$.

Thus, if $\boldsymbol{n = 4k - 3}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, **sin** $\boldsymbol{\{n\pi/2 + (-1)^n(\pi/6)\} =}$ **sin** $\boldsymbol{\pi/3 = \frac{\sqrt{3}}{2}}$.

So putting together now, the case where $\boldsymbol{k}$ is greater than or equal to 0, and the case where $\boldsymbol{k}$ is less than 0, we can put them the way below.

If $\boldsymbol{n = 4k}$ for $\boldsymbol{k}$ integer, $\mathbf{sin}\left(\frac{\boldsymbol{n}\pi}{2} + (-1)^{\boldsymbol{n}}\frac{\pi}{6}\right) = \frac{1}{2}$.

If $\boldsymbol{n = 4k + 1}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, $\mathbf{sin}\left(\frac{\boldsymbol{n}\pi}{2} + (-1)^{\boldsymbol{n}}\frac{\pi}{6}\right) = \frac{\sqrt{\mathbf{3}}}{\mathbf{2}}$.

If $\boldsymbol{n = 4k - 3}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, $\mathbf{sin}\left(\frac{\boldsymbol{n}\pi}{2} + (-1)^{\boldsymbol{n}}\frac{\pi}{6}\right) = \frac{\sqrt{\mathbf{3}}}{\mathbf{2}}$.

If $\boldsymbol{n = 4k + 3}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, $\mathbf{sin}\left(\frac{\boldsymbol{n}\pi}{2} + (-1)^{\boldsymbol{n}}\frac{\pi}{6}\right) = -\frac{\sqrt{\mathbf{3}}}{\mathbf{2}}$.

If $\boldsymbol{n = 4k - 1}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, $\mathbf{sin}\left(\frac{\boldsymbol{n}\pi}{2} + (-1)^{\boldsymbol{n}}\frac{\pi}{6}\right) = -\frac{\sqrt{\mathbf{3}}}{\mathbf{2}}$.

If $\boldsymbol{n = 4k + 2}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, $\mathbf{sin}\left(\frac{\boldsymbol{n}\pi}{2} + (-1)^{\boldsymbol{n}}\frac{\pi}{6}\right) = -\frac{1}{2}$.

And if $\boldsymbol{n = 4k - 2}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, $\mathbf{sin}\left(\frac{\boldsymbol{n}\pi}{2} + (-1)^{\boldsymbol{n}}\frac{\pi}{6}\right) = -\frac{1}{2}$.

Taking a closer look at however, for instance, the case where $\boldsymbol{n = 4k + 1}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, and the case where $\boldsymbol{n = 4k - 3}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, we can just put both together the way as follows: $\boldsymbol{n = 4k + 1}$ for $\boldsymbol{k}$ integer. How?

If $\boldsymbol{n = 4k + 1}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, we get $\boldsymbol{n}$ = 1, 5, 9, 13, 17, and so on.

If $\boldsymbol{n = 4k - 3}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, we get $\boldsymbol{n}$ = -3, -7, -11, -15, and so forth.

And if $\boldsymbol{n = 4k + 1}$ for $\boldsymbol{k}$ integer, that is, $\boldsymbol{k}$ = 0, 1, -1, 2, -2, 3, -3, 4, -4, etc., we get $\boldsymbol{n}$ = 1, 5, -3, 9, -7, 13, -11, 17, -15, …

And the same is true for the other cases where we get the same ratios.
So we can put together all the cases above the way below:

If $\boldsymbol{n = 4k}$ for $\boldsymbol{k}$ integer, $\mathbf{sin}\left(\frac{\boldsymbol{n}\pi}{2}+(-1)^{\boldsymbol{n}}\frac{\pi}{6}\right)=\frac{1}{2}$.

If $\boldsymbol{n = 4k + 1}$ for $\boldsymbol{k}$ integer, $\mathbf{sin}\left(\frac{\boldsymbol{n}\pi}{2}+(-1)^{\boldsymbol{n}}\frac{\pi}{6}\right)=\frac{\sqrt{\mathbf{3}}}{\mathbf{2}}$.

If $\boldsymbol{n = 4k + 2}$ for $\boldsymbol{k}$ integer, $\mathbf{sin}\left(\frac{\boldsymbol{n}\pi}{2}+(-1)^{\boldsymbol{n}}\frac{\pi}{6}\right)=-\frac{1}{2}$.

And if $\boldsymbol{n = 4k + 3}$ for $\boldsymbol{k}$ integer, $\mathbf{sin}\left(\frac{\boldsymbol{n}\pi}{2}+(-1)^{\boldsymbol{n}}\frac{\pi}{6}\right)=-\frac{\sqrt{\mathbf{3}}}{\mathbf{2}}$.

And in fact, we can put together all the cases above the way below, too.

If $\boldsymbol{n = 4k}$ for $\boldsymbol{k}$ integer, $\mathbf{sin}\left(\frac{\boldsymbol{n}\pi}{2}+(-1)^{\boldsymbol{n}}\frac{\pi}{6}\right)=\frac{1}{2}$.

If $\boldsymbol{n = 4k - 1}$ for $\boldsymbol{k}$ integer, $\mathbf{sin}\left(\frac{\boldsymbol{n}\pi}{2}+(-1)^{\boldsymbol{n}}\frac{\pi}{6}\right)=-\frac{\sqrt{\mathbf{3}}}{\mathbf{2}}$.

if $\boldsymbol{n = 4k - 2}$ for $\boldsymbol{k}$ integer, $\mathbf{sin}\left(\frac{\boldsymbol{n}\pi}{2}+(-1)^{\boldsymbol{n}}\frac{\pi}{6}\right)=-\frac{1}{2}$.

And if $\boldsymbol{n = 4k - 3}$ for $\boldsymbol{k}$ integer, $\mathbf{sin}\left(\frac{\boldsymbol{n}\pi}{2}+(-1)^{\boldsymbol{n}}\frac{\pi}{6}\right)=\frac{\sqrt{\mathbf{3}}}{\mathbf{2}}$. How?

If $\boldsymbol{n} = \mathbf{4}\boldsymbol{k} + \mathbf{1}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, we get $\boldsymbol{n} = 1, 5, 9, 13$, and so on.

If $\boldsymbol{n} = \mathbf{4}\boldsymbol{k} - \mathbf{3}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, we get $\boldsymbol{n} = -3, -7, -11, -15, -19$, and so forth.

And if $\boldsymbol{n} = \mathbf{4}\boldsymbol{k} - \mathbf{3}$ for $\boldsymbol{k}$ integer, that is, $\boldsymbol{k} = 0, 1, -1, 2, -2, 3, -3, 4, -4$, etc., we get: $\boldsymbol{n} = -3, 1, -7, 5, -11, 9, -15, 13, -19, \ldots$

And the same is true for the other cases where we get the same ratios.

So putting threads together, we want to consider the four cases where $\boldsymbol{n} = \mathbf{4}\boldsymbol{k}$, $\mathbf{4}\boldsymbol{k} + \mathbf{1}$, $\mathbf{4}\boldsymbol{k} + \mathbf{2}$, and $\mathbf{4}\boldsymbol{k} + \mathbf{3}$ for $\boldsymbol{k}$ integer.