Examples 2 in Coordinate Trigonometry

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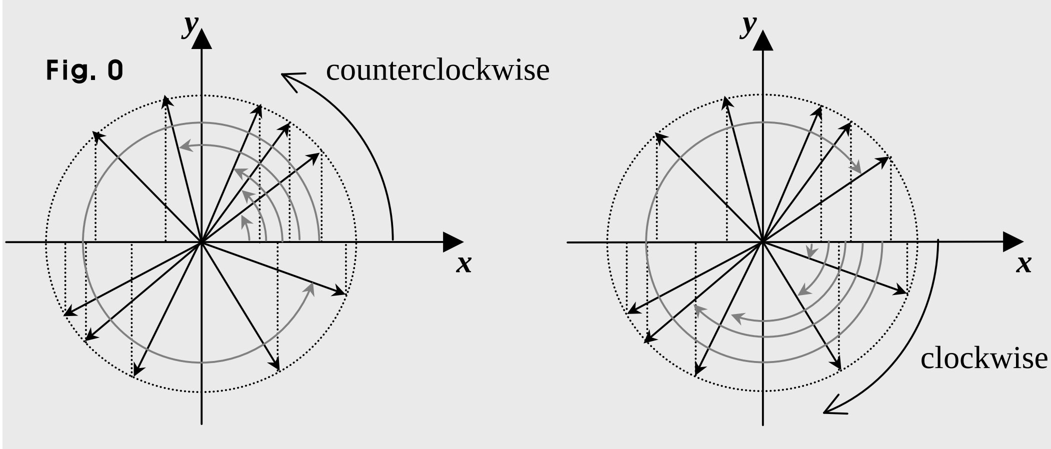
Fig. 0



In coordinate trigonometry, the vector turning makes the angles.
If turning clockwise, it makes angles negative, and if counterclockwise, it makes angles positive. And of course, if no turning, the angle made is 0.
So the angle can be 0^{o}, positive, or negative.

In each right triangle, the vector is of length 1, and is the hypotenuse.
And (x, y) is the terminal point, so x is the adjacent, and y is the opposite.

Thus, assuming θ is the angle, we get $\sin\theta = y$, $\cos\theta = x$, and $\tan\theta = \frac{y}{x}$.

So if the vector is in the first quadrant, all the three trig-ratios > **0**, since x and y both > **0**.
In the second, only the sines > **0**, since $x < 0$, and $y > 0$.
In the third, only the tangents > **0**, since $x < 0$, and $y < 0$.
And in the fourth, only the cosines > **0**, since $x > 0$, and $y < 0$.

0. Find the value of $\sin\{5n\pi/2 + (-1)^n(\pi/6)\}$ where n is a positive integer.

1. Find the value of $\sin\{5n\pi + (-1)^n(\pi/6)\}$ where n is a nonnegative integer.

Suggestions or Solutions
To the Problem in the Example 0

Find the value of sin $\{5n\pi/2 + (-1)^n(\pi/6)\}$ where n is a positive integer.

The angle depends on the value of n.
So let's first, put some values into n beginning with 1, and see how the angle changes.

- $n = 1 \Rightarrow 5n\pi/2 + (-1)^n(\pi/6) = 5\pi/2 - \pi/6 = 2\pi + \pi/3$.

$n = 2 \Rightarrow 5n\pi/2 + (-1)^n(\pi/6) = 10\pi/2 + \pi/6 = 4\pi + \pi + \pi/6$.

$n = 3 \Rightarrow 5n\pi/2 + (-1)^n(\pi/6) = 15\pi/2 - \pi/6 = 7\pi + \pi/2 - \pi/6 = 6\pi + \pi + \pi/3$.

$n = 4 \Rightarrow 5n\pi/2 + (-1)^n(\pi/6) = 20\pi/2 + \pi/6 = 10\pi + \pi/6$.

- $n = 5 \Rightarrow 5n\pi/2 + (-1)^n(\pi/6) = 25\pi/2 - \pi/6 = 12\pi + \pi/2 - \pi/6 = 12\pi + \pi/3$.

$n = 6 \Rightarrow 5n\pi/2 + (-1)^n(\pi/6) = 30\pi/2 + \pi/6 = 14\pi + \pi + \pi/6$.

$n = 7 \Rightarrow 5n\pi/2 + (-1)^n(\pi/6) = 35\pi/2 - \pi/6 = 17\pi + \pi/2 - \pi/6 = 16\pi + \pi + \pi/3$.

$n = 8 \Rightarrow 5n\pi/2 + (-1)^n(\pi/6) = 40\pi/2 + \pi/6 = 20\pi + \pi/6$.

- $n = 9 \Rightarrow 5n\pi/2 + (-1)^n(\pi/6) = 45\pi/2 - \pi/6 = 22\pi + \pi/2 - \pi/6 = 22\pi + \pi/3$.

$n = 10 \Rightarrow 5n\pi/2 + (-1)^n(\pi/6) = 50\pi/2 + \pi/6 = 24\pi + \pi + \pi/6$.

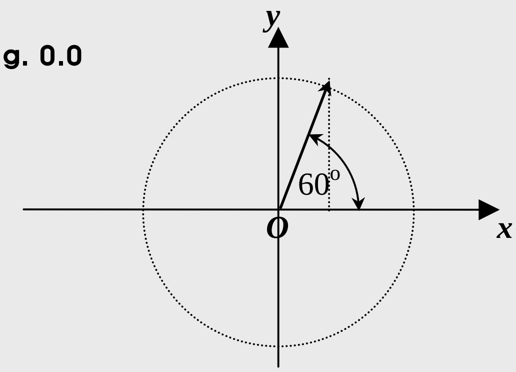
$n = 11 \Rightarrow 5n\pi/2 + (-1)^n(\pi/6) = 55\pi/2 - \pi/6 = 26\pi + \pi + \pi/2 - \pi/6 = 26\pi + \pi + \pi/3$.

$n = 12 \Rightarrow 5n\pi/2 + (-1)^n(\pi/6) = 60\pi/2 + \pi/6 = 30\pi + \pi/6$.

- $n = 13 \Rightarrow 5n\pi/2 + (-1)^n(\pi/6) = 65\pi/2 - \pi/6 = 32\pi + \pi/3$.

Now, taking a closer look at the sequence above, we can notice that if $\boldsymbol{n = 4k + 1}$ for $\boldsymbol{k}$ integer ≥ **0**, all the vectors get the same spot in the first quadrant.

Fig. 0.0



Thus, if $\boldsymbol{n = 4k + 1}$ for $\boldsymbol{k}$ integer ≥ **0**, the sine of $\boldsymbol{\{5n\pi/2 + (-1)^n(\pi/6)\}}$ is the same, and is the same as $\boldsymbol{\sin \pi/3}$, which is $\frac{\sqrt{3}}{2}$.

So if $\boldsymbol{n = 4k + 1}$ for $\boldsymbol{k}$ integer ≥ **0**, $\boldsymbol{\sin \{5n\pi/2 + (-1)^n(\pi/6)\} = \sin \pi/3 =} \frac{\sqrt{3}}{2}$.

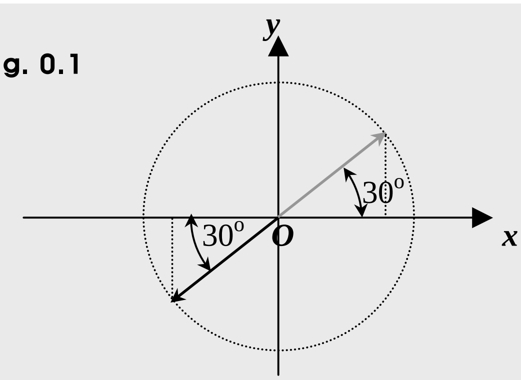
Next, we can notice that if $\boldsymbol{n = 4k + 2}$ for $\boldsymbol{k}$ integer ≥ **0**, all the vectors get the same spot again.

$\boldsymbol{n = 2 \Rightarrow 5n\pi/2 + (-1)^n(\pi/6) = 10\pi/2 + \pi/6 = 4\pi + \pi + \pi/6}$.

$\boldsymbol{n = 6 \Rightarrow 5n\pi/2 + (-1)^n(\pi/6) = 30\pi/2 + \pi/6 = 14\pi + \pi + \pi/6}$.

$\boldsymbol{n = 10 \Rightarrow 5n\pi/2 + (-1)^n(\pi/6) = 50\pi/2 + \pi/6 = 24\pi + \pi + \pi/6}$.

Fig. 0.1



So if $\boldsymbol{n = 4k + 2}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, all the vectors get the same spot in the third quadrant.

Thus, if $\boldsymbol{n = 4k + 2}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, the sine of $\mathbf{\{5n\pi/2 + (-1)^n(\pi/6)\}}$ is the same, and is the same as $\mathbf{-sin\ (\pi/6)}$, which is -1/2.

So if $\boldsymbol{n = 4k + 2}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, $\mathbf{sin\ \{5n\pi/2 + (-1)^n(\pi/6)\} = -sin\ \pi/6 = -1/2}$.

Next, we can notice that if $\boldsymbol{n = 4k + 3}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, all the vectors get the same spot again.

$\boldsymbol{n = 3 \Rightarrow 5n\pi/2 + (-1)^n(\pi/6) = 15\pi/2 - \pi/6 = 7\pi + \pi/2 - \pi/6 = 6\pi + \pi + \pi/3}$.

$\boldsymbol{n = 7 \Rightarrow 5n\pi/2 + (-1)^n(\pi/6) = 35\pi/2 - \pi/6 = 17\pi + \pi/2 - \pi/6 = 16\pi + \pi + \pi/3}$.

$\boldsymbol{n = 11 \Rightarrow 5n\pi/2 + (-1)^n(\pi/6) = 55\pi/2 - \pi/6 = 26\pi + \pi + \pi/2 - \pi/6 = 26\pi + \pi + \pi/3}$.

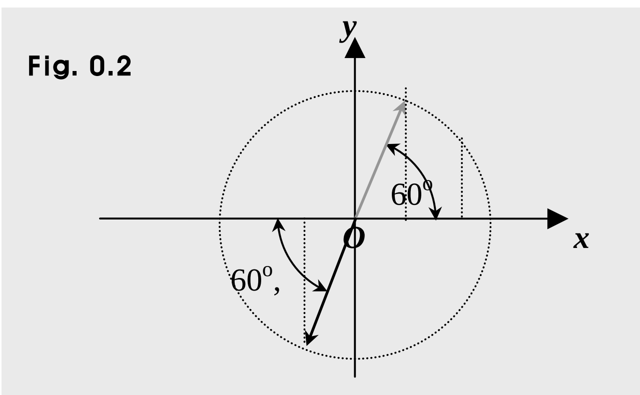


Fig. 0.2

So if $\boldsymbol{n = 4k + 3}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, all the vectors get the same spot in the third quadrant.

Thus, if $\boldsymbol{n = 4k + 3}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$ the sine of $\mathbf{\{5n\pi/2 + (-1)^n(\pi/6)\}}$ is the same, and is the same as $\mathbf{sin\ (\pi + \pi/3)}$, which is $-\frac{\sqrt{3}}{2}$.

That's because $\mathbf{sin\ (\pi + \pi/6) = -sin\ \pi/3} = -\frac{\sqrt{3}}{2}$.

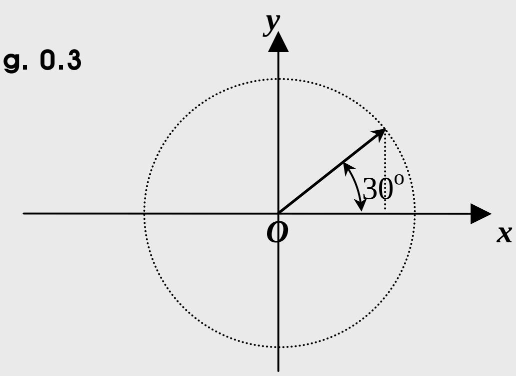
So if $\boldsymbol{n = 4k + 3}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, $\mathbf{sin\ \{5n\pi/2 + (-1)^n(\pi/6)\} = -sin\ \pi/3} = -\frac{\sqrt{3}}{2}$.
And next, we can notice that if $\boldsymbol{n = 4k + 4 = 4(k + 1)}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, all the vectors get the same spot again.

$\boldsymbol{n = 4 \Rightarrow 5n\pi/2 + (-1)^n(\pi/6) = 20\pi/2 + \pi/6 = 10\pi + \pi/6}$.

$\boldsymbol{n = 8 \Rightarrow 5n\pi/2 + (-1)^n(\pi/6) = 40\pi/2 + \pi/6 = 20\pi + \pi/6}$.

$\boldsymbol{n = 12 \Rightarrow 5n\pi/2 + (-1)^n(\pi/6) = 60\pi/2 + \pi/6 = 30\pi + \pi/6}$.

Fig. 0.3



So if $\boldsymbol{n = 4(k + 1)}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, all the vectors get the same spot in the first quadrant. Thus, if $\boldsymbol{n = 4(k + 1)}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, the sine of $\boldsymbol{5n\pi/2 + (-1)^n(\pi/6)}$ is the same, and is the same as $\mathbf{sin}\,\frac{\pi}{6}$, which is $\frac{1}{2}$.

So if $\boldsymbol{n = 4(k + 1)}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, $\mathbf{sin}\left\{\frac{5\boldsymbol{n}\pi}{2} + (-1)^{\boldsymbol{n}}\frac{\pi}{6}\right\} = \mathbf{sin}\,\frac{\pi}{6} = \frac{1}{2}$.

So putting together now, all the four cases above, we can put them the way as follows.

If $\boldsymbol{n = 4k + 1}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, $\mathbf{sin}\left\{\frac{5\boldsymbol{n}\pi}{2} + (-1)^{\boldsymbol{n}}\frac{\pi}{6}\right\} = \mathbf{sin}\,\frac{\pi}{3} = \frac{\sqrt{\mathbf{3}}}{\mathbf{2}}$

If $\boldsymbol{n = 4k + 2}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, $\mathbf{sin}\left\{\frac{5\boldsymbol{n}\pi}{2} + (-1)^{\boldsymbol{n}}\frac{\pi}{6}\right\} = -\mathbf{sin}\,\frac{\pi}{6} = -\frac{\mathbf{1}}{\mathbf{2}}$.

If $\boldsymbol{n = 4k + 3}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, $\mathbf{sin}\left\{\frac{5\boldsymbol{n}\pi}{2} + (-1)^{\boldsymbol{n}}\frac{\pi}{6}\right\} = -\mathbf{sin}\,\frac{\pi}{3} = -\frac{\sqrt{\mathbf{3}}}{\mathbf{2}}$

And if $\boldsymbol{n = 4(k + 1)}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, $\mathbf{sin}\left\{\frac{5\boldsymbol{n}\pi}{2} + (-1)^{\boldsymbol{n}}\frac{\pi}{6}\right\} = \mathbf{sin}\,\frac{\pi}{6} = \frac{\mathbf{1}}{\mathbf{2}}$.

Suggestions or Solutions
To the Problem in the Example 1

Find the value of sin $\{5n\pi + (-1)^n(\pi/6)\}$ where n is a nonnegative integer.

The angle depends on the value of ***n***.
So let's first, put some values into ***n*** beginning with 0, and see how the angle changes.

- $n = 0 \Rightarrow 5n\pi + (-1)^n(\pi/6) = \pi/6.$

$n = 1 \Rightarrow 5n\pi + (-1)^n(\pi/6) = 5\pi - \pi/6 = 4\pi + 5\pi/6.$

- $n = 2 \Rightarrow 5n\pi + (-1)^n(\pi/6) = 10\pi + \pi/6.$

$n = 3 \Rightarrow 5n\pi + (-1)^n(\pi/6) = 15\pi - \pi/6 = 14\pi + 5\pi/6.$

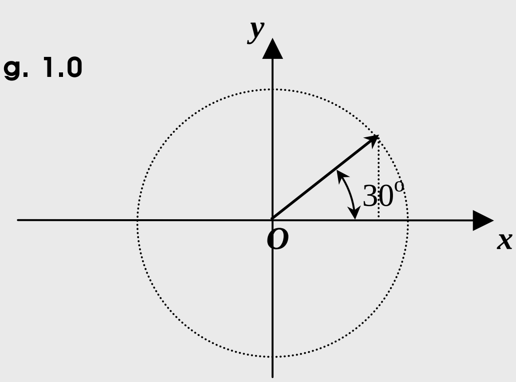
- $n = 4 \Rightarrow 5n\pi + (-1)^n(\pi/6) = 20\pi + \pi/6.$

$n = 5 \Rightarrow 5n\pi + (-1)^n(\pi/6) = 25\pi - \pi/6 = 24\pi + 5\pi/6.$

- $n = 6 \Rightarrow 5n\pi + (-1)^n(\pi/6) = 30\pi + \pi/6.$

Now, we can notice that if $n = 2k$ for k integer ≥ 0, all the vectors get the same spot.

Fig. 1.0



So if $n = 2k$ for k integer ≥ 0, all the vectors get the same spot in the first quadrant.

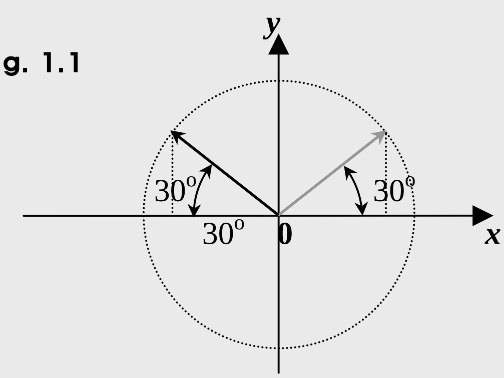
Thus, if $\boldsymbol{n} = \mathbf{2}\boldsymbol{k}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, the sine of $\{5n\pi + (-1)^n(\pi/6)\}$ is the same, and is the same as $\sin \pi/6$, which is 1/2.
So if $n = 2k$ for k integer ≥ 0, $\sin \{5n\pi + (-1)^n(\pi/6)\} = \sin \pi/6 = 1/2$.
Next, we can notice that if $n = 2k + 1$ for k integer ≥ 0, all the vectors get the same spot again.

$n = 1 \Rightarrow 5n\pi + (-1)^n(\pi/6) = 5\pi - \pi/6 = 4\pi + 5\pi/6$.

$n = 3 \Rightarrow 5n\pi + (-1)^n(\pi/6) = 15\pi - \pi/6 = 14\pi + 5\pi/6$.

$n = 5 \Rightarrow 5n\pi + (-1)^n(\pi/6) = 25\pi - \pi/6 = 24\pi + 5\pi/6$.

Fig. 1.1



So if $n = 2k + 1$ for k integer ≥ 0, all the vectors get the same spot in the second quadrant.
Thus, if $n = 2k + 1$ for k integer ≥ 0, the sine of $\{5n\pi + (-1)^n(\pi/6)\}$ is the same, and is the same as $\sin (5\pi/6)$, which is 1/2. That's because $\sin (5\pi/6) = \sin \pi/6$

So if $n = 2k + 1$ for k integer ≥ 0, $\sin \{5n\pi + (-1)^n(\pi/6)\} = \sin 5\pi/6 = \sin \pi/6 = 1/2$.

Thus, putting together now, the two cases above, we can put them the way as follows.

If $n = 2k$ for k integer ≥ 0, $\sin (5n\pi + (-1)^n \frac{\pi}{6}) = \frac{1}{2}$.

Also, if $n = 2k + 1$ for k integer ≥ 0, $\sin (5n\pi + (-1)^n \frac{\pi}{6}) = \frac{1}{2}$.

Thus, we get $\sin (5n\pi + (-1)^n \frac{\pi}{6}) = \frac{1}{2}$ for n nonnegative integer.