Examples 3 in Coordinate Trigonometry

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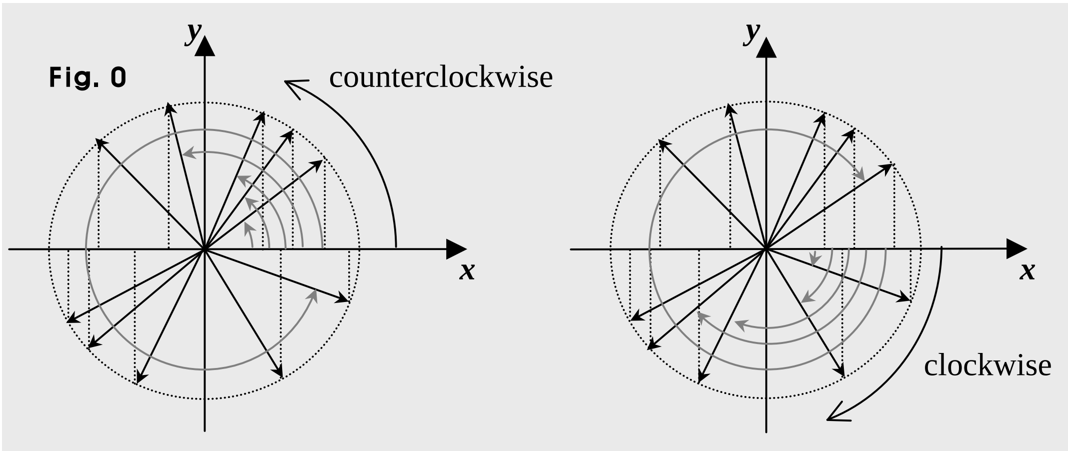
Fig. 0



In coordinate trigonometry, the vector turning makes the angles.
If turning clockwise, it makes angles negative, and if counterclockwise, it makes angles positive. And of course, if no turning, the angle made is 0.
So the angle can be 0^o or any angle positive or negative.

In each right triangle, the vector is of length 1, and is the hypotenuse.
And $\boldsymbol{(x, y)}$ is the terminal point, so $\boldsymbol{x}$ is the adjacent, and $\boldsymbol{y}$ is the opposite.

Thus, assuming θ is the angle, we get $\mathbf{sin}\ \theta = y$, $\mathbf{cos}\ \theta = x$, and $\mathbf{tan}\ \theta = \dfrac{y}{x}$.

So if the vector is in the first quadrant, all the three trig ratios > **0**, since $\boldsymbol{x}$ and $\boldsymbol{y}$ both > **0**.
In the second, only the sines > **0**, since $x < 0$, and $y > 0$.
In the third, only the tangents > **0**, since $x < 0$, and $y < 0$.
And in the fourth, only the cosines > **0**, since $x > 0$, and $y < 0$.

0. Find the value of $\mathbf{cos}\ \{n\pi + (-1)^n(\pi/6)\}$ where n is an integer.

1. Find the value of $\mathbf{cos}\ \{n\pi/2 + (-1)^n(\pi/6)\}$ where n is an integer.

Suggestions or Solutions
To the **Problem** in the Example 0

Find the value of cos $\{n\pi + (-1)^n(\pi/6)\}$ where n is an integer.

The angle depends on the value of n.
So let's first, put some values into n beginning with 0, and see how the angle changes.

• $n = 0 \Rightarrow n\pi + (-1)^n(\pi/6) = \pi/6$.

$n = 1 \Rightarrow n\pi + (-1)^n(\pi/6) = \pi - \pi/6 = 5\pi/6$.

• $n = 2 \Rightarrow n\pi + (-1)^n(\pi/6) = 2\pi + \pi/6$.

$n = 3 \Rightarrow n\pi + (-1)^n(\pi/6) = 3\pi - \pi/6 = 2\pi + 5\pi/6$.

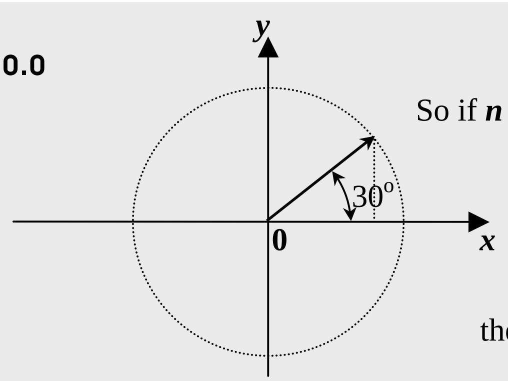
• $n = 4 \Rightarrow n\pi + (-1)^n(\pi/6) = 4\pi + \pi/6$.

$n = 5 \Rightarrow n\pi + (-1)^n(\pi/6) = 5\pi - \pi/6 = 4\pi + 5\pi/6$.

• $n = 6 \Rightarrow n\pi + (-1)^n(\pi/6) = 6\pi + \pi/6$.

Now, we can notice that if $n = 2k$ for k integer ≥ 0, all the vectors get the same spot.

Fig. 0.0



So if $n = 2k$ for k integer ≥ 0, all the vectors get the same spot in the first quadrant.

Thus, if $n = 2k$ for k integer ≥ 0, the cosine of $\{n\pi + (-1)^n(\pi/6)\}$ is the same, and is the same as $\cos \pi/6$, which is $\frac{\sqrt{3}}{2}$.

So if $n = 2k$ for k integer ≥ 0, $\cos \{n\pi + (-1)^n(\pi/6)\} = \cos \pi/6 = \frac{\sqrt{3}}{2}$.

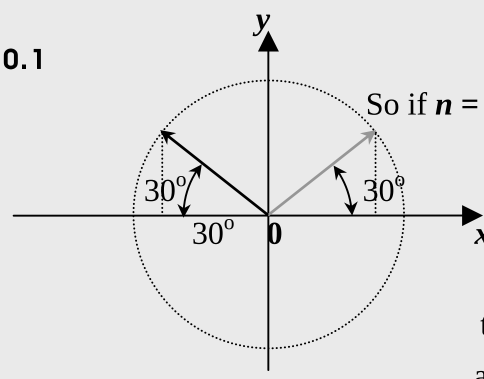
Next, we can notice that if $\boldsymbol{n = 2k + 1}$ for $\boldsymbol{k}$ integer $\geq$ **0**, all the vectors get the same spot again.

$\boldsymbol{n = 1 \Rightarrow n\pi + (-1)^n(\pi/6) = \pi - \pi/6 = 5\pi/6}$.

$\boldsymbol{n = 3 \Rightarrow n\pi + (-1)^n(\pi/6) = 3\pi - \pi/6 = 2\pi + 5\pi/6}$.

$\boldsymbol{n = 5 \Rightarrow n\pi + (-1)^n(\pi/6) = 5\pi - \pi/6 = 4\pi + 5\pi/6}$.

Fig. 0.1



So if $\boldsymbol{n = 2k + 1}$ for $\boldsymbol{k}$ integer $\geq$ **0**, all the vectors get the same spot in the second quadrant.

Thus, if $\boldsymbol{n = 2k + 1}$ for $\boldsymbol{k}$ integer $\geq$ **0**, the cosine of $\boldsymbol{\{n\pi + (-1)^n(\pi/6)\}}$ is the same, and is the same as **cos (5π/6)**, which is $-\frac{\sqrt{3}}{2}$.

That's because **cos (5π/6) = -cos π/6**

So if $\boldsymbol{n = 2k + 1}$ for $\boldsymbol{k}$ integer $\geq$ **0**, $\boldsymbol{\cos \{n\pi + (-1)^n(\pi/6)\} = \cos 5\pi/6 = -\cos \pi/6} = -\frac{\sqrt{3}}{2}$.

So putting together now, the two cases above, we can put them the way below.

If $\boldsymbol{n = 2k}$ for $\boldsymbol{k}$ integer $\geq$ **0**, $\boldsymbol{\cos \{n\pi + (-1)^n(\pi/6)\}} = \frac{\sqrt{3}}{2}$.

And if $\boldsymbol{n = 2k + 1}$ for $\boldsymbol{k}$ integer $\geq$ **0**, $\boldsymbol{\cos \{n\pi + (-1)^n(\pi/6)\}} = -\frac{\sqrt{3}}{2}$.

Next, putting some negative values into $\boldsymbol{n}$ beginning with 0, we get a sequence as follows.

• $n = 0 \Rightarrow n\pi + (-1)^n(\pi/6) = \pi/6$.

$n = -1 \Rightarrow n\pi + (-1)^n(\pi/6) = -\pi - \pi/6$.

• $n = -2 \Rightarrow n\pi + (-1)^n(\pi/6) = -2\pi + \pi/6$.

$n = -3 \Rightarrow n\pi + (-1)^n(\pi/6) = -3\pi - \pi/6 = -2\pi - \pi - \pi/6$.

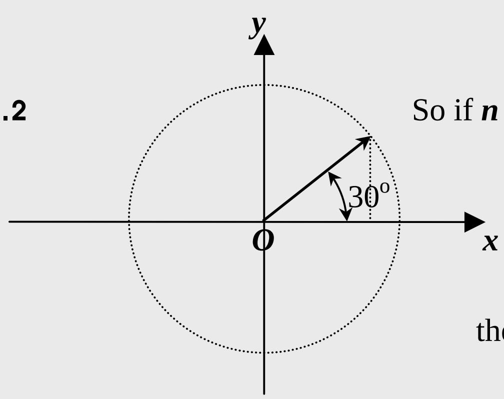
• $n = -4 \Rightarrow n\pi + (-1)^n(\pi/6) = -4\pi + \pi/6$.

$n = -5 \Rightarrow n\pi + (-1)^n(\pi/6) = -5\pi - \pi/6 = -4\pi - \pi - \pi/6$.

• $n = -6 \Rightarrow n\pi + (-1)^n(\pi/6) = -6\pi + \pi/6$.

Now, we can notice that if $n = 2k$ for k integer ≤ 0, all the vectors get the same spot again.

Fig. 0.2



So if $n = 2k$ for k integer ≥ 0, all the vectors get the same spot in the first quadrant.

Thus, if $n = 2k$ for k integer ≥ 0, the cosine of $\{n\pi + (-1)^n(\pi/6)\}$ is the same, and is the same as $\cos \pi/6$, which is $\frac{\sqrt{3}}{2}$.

So if $n = 2k$ for k integer ≤ 0, $\cos \{n\pi + (-1)^n(\pi/6)\} = \cos \pi/6 = \frac{\sqrt{3}}{2}$.

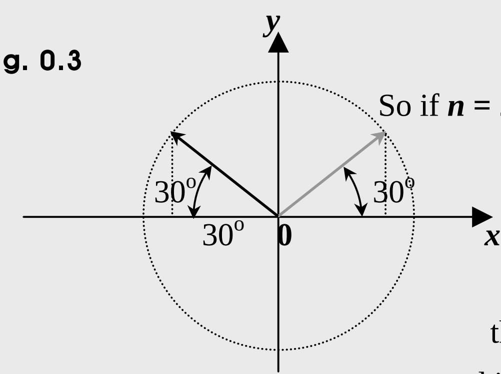
Next, we can notice that if $n = 2k - 1$ for k integer ≤ 0, all the vectors get the same spot again.

$n = -1 \Rightarrow n\pi + (-1)^n(\pi/6) = -\pi - \pi/6$.

$n = -3 \Rightarrow n\pi + (-1)^n(\pi/6) = -3\pi - \pi/6 = -2\pi - \pi - \pi/6$.

$n = -5 \Rightarrow n\pi + (-1)^n(\pi/6) = -5\pi - \pi/6 = -4\pi - \pi - \pi/6$.

Fig. 0.3



So if $n = 2k - 1$ for k integer ≤ 0, all the vectors get the same spot in the second quadrant.

Thus, if $n = 2k - 1$ for k integer ≤ 0, the cosine of $\{n\pi + (-1)^n(\pi/6)\}$ is the same, and is the same as $\cos(-\pi - \pi/6)$, which is $-\frac{\sqrt{3}}{2}$.

That's because $\cos(-\pi - \pi/6) = -\cos \pi/6$

So if $n = 2k - 1$ for k integer ≥ 0, $\cos\{n\pi + (-1)^n(\pi/6)\} = \cos(-\pi - \pi/6) = -\cos \pi/6 = -\frac{\sqrt{3}}{2}$.

So putting together now, the two cases above, we can put them the way below.

If $n = 2k$ for k integer ≤ 0, $\cos\{n\pi + (-1)^n(\pi/6)\} = \frac{\sqrt{3}}{2}$.
And if $n = 2k - 1$ for k integer ≤ 0, $\cos\{n\pi + (-1)^n(\pi/6)\} = -\frac{\sqrt{3}}{2}$.

Also, we have these:
If $n = 2k$ for k integer ≥ 0, $\cos\{n\pi + (-1)^n(\pi/6)\} = \frac{\sqrt{3}}{2}$.
And if $n = 2k + 1$ for k integer ≥ 0, $\cos\{n\pi + (-1)^n(\pi/6)\} = -\frac{\sqrt{3}}{2}$.

And taking a closer look at the case where $n = 2k - 1$ for k integer ≤ 0, and the case where $n = 2k + 1$ for k integer ≥ 0, we can just put both together the way as follows.

$n = 2k + 1$ for k integer   How?

If $\boldsymbol{n} = \mathbf{2}\boldsymbol{k} + \mathbf{1}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, we get $\boldsymbol{n} = 1, 3, 5, 7, 9$, and so on.

If $\boldsymbol{n} = \mathbf{2}\boldsymbol{k} - \mathbf{1}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, we get $\boldsymbol{n} = -1, -3, -5, -7$, and so forth.

And if $\boldsymbol{n} = \mathbf{2}\boldsymbol{k} + \mathbf{1}$ for $\boldsymbol{k}$ integer, that is, $\boldsymbol{k} = 0, 1, -1, 2, -2, 3, -3, 4, -4$, etc., we get $\boldsymbol{n} = 1, 3, -1, 5, -3, 7, -5, 9, -7, \ldots$

So putting threads together, we get these:

If $\boldsymbol{n} = \mathbf{2}\boldsymbol{k}$ for $\boldsymbol{k}$ integer, $\mathbf{cos}\ \{\boldsymbol{n}\pi + (\mathbf{-1})^{\boldsymbol{n}}(\pi/\mathbf{6})\} = \frac{\sqrt{3}}{2}$.

And if $\boldsymbol{n} = \mathbf{2}\boldsymbol{k} + \mathbf{1}$ for $\boldsymbol{k}$ integer, $\mathbf{cos}\ \{\boldsymbol{n}\pi + (\mathbf{-1})^{\boldsymbol{n}}(\pi/\mathbf{6})\} = -\frac{\sqrt{3}}{2}$.

And we can put it the way below, too.

If $\boldsymbol{n} = \mathbf{2}\boldsymbol{k}$ for $\boldsymbol{k}$ integer, $\mathbf{cos}\ \{\boldsymbol{n}\pi + (\mathbf{-1})^{\boldsymbol{n}}(\pi/\mathbf{6})\} = \frac{\sqrt{3}}{2}$.

And if $\boldsymbol{n} = \mathbf{2}\boldsymbol{k} - \mathbf{1}$ for $\boldsymbol{k}$ integer, $\mathbf{cos}\ \{\boldsymbol{n}\pi + (\mathbf{-1})^{\boldsymbol{n}}(\pi/\mathbf{6})\} = -\frac{\sqrt{3}}{2}$.

Suggestions or Solutions

To the Problem in the Example 1

Find the value of cos $\{n\pi/2 + (-1)^n(\pi/6)\}$ where n is an integer.

The angle depends on the value of n.
So let's first, put some values into n beginning with 0, and see how the angle changes.

- $n = 0 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = \pi/6$.

$n = 1 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = \pi/2 - \pi/6 = \pi/3$.

$n = 2 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 2\pi/2 + \pi/6 = \pi + \pi/6$.

$n = 3 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 3\pi/2 - \pi/6 = \pi + \pi/2 - \pi/6 = \pi + \pi/3$.

- $n = 4 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 4\pi/2 + \pi/6 = 2\pi + \pi/6$.

$n = 5 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 5\pi/2 - \pi/6 = 2\pi + \pi/2 - \pi/6 = 2\pi + \pi/3$.

$n = 6 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 6\pi/2 + \pi/6 = 2\pi + \pi + \pi/6$.

$n = 7 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 7\pi/2 - \pi/6 = 2\pi + 3\pi/2 - \pi/6 = 2\pi + \pi + \pi/3$.

- $n = 8 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 8\pi/2 + \pi/6 = 4\pi + \pi/6$.

$n = 9 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 9\pi/2 - \pi/6 = 4\pi + \pi/2 - \pi/6 = 4\pi + \pi/3$.

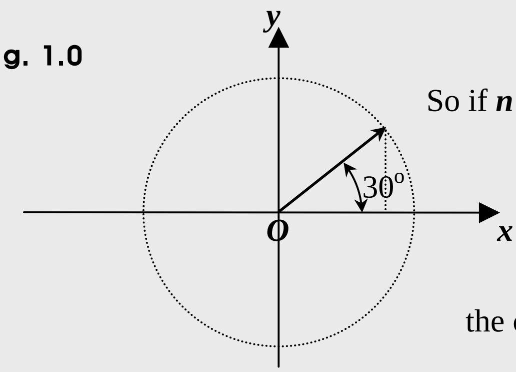
$n = 10 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 10\pi/2 + \pi/6 = 4\pi + \pi + \pi/6$.

$n = 11 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 11\pi/2 - \pi/6 = 4\pi + 3\pi/2 - \pi/6 = 4\pi + \pi + \pi/3$.

- $n = 12 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 12\pi/2 + \pi/6 = 6\pi + \pi/6$.

Now, taking a closer look at the sequence of angles above, we can notice that if $\boldsymbol{n = 4k}$ for $\boldsymbol{k}$ integer $\geq$ **0**, all the vectors get the same spot.

Fig. 1.0



So if $\boldsymbol{n = 4k}$ for $\boldsymbol{k}$ integer $\geq$ **0**, all the vectors get the same spot in the first quadrant.

Thus, if $\boldsymbol{n = 4k}$ for $\boldsymbol{k}$ integer $\geq$ **0**, the cosine of $\boldsymbol{\{n\pi/2 + (-1)^n(\pi/6)\}}$ is the same, and is the same as $\boldsymbol{\cos \pi/6}$, which is $\frac{\sqrt{3}}{2}$.

So if $\boldsymbol{n = 4k}$ for $\boldsymbol{k}$ integer $\geq$ **0**, $\boldsymbol{\cos \{n\pi/2 + (-1)^n(\pi/6)\} = \cos \pi/6} = \frac{\sqrt{3}}{2}$.

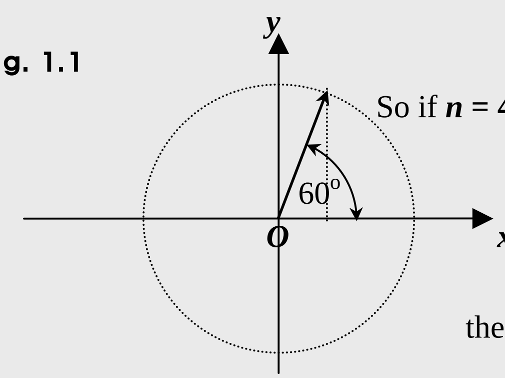
Next, we can notice that if $\boldsymbol{n = 4k + 1}$ for $\boldsymbol{k}$ integer $\geq$ **0**, all the vectors get the same spot again.

$\boldsymbol{n = 1 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = \pi/2 - \pi/6 = \pi/3}$.

$\boldsymbol{n = 5 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 5\pi/2 - \pi/6 = 2\pi + \pi/2 - \pi/6 = 2\pi + \pi/3}$.

$\boldsymbol{n = 9 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = 9\pi/2 - \pi/6 = 4\pi + \pi/2 - \pi/6 = 4\pi + \pi/3}$.

Fig. 1.1



So if $\boldsymbol{n = 4k + 1}$ for $\boldsymbol{k}$ integer $\geq$ **0**, all the vectors get the same spot in the first quadrant.

Thus, if $\boldsymbol{n = 4k + 1}$ for $\boldsymbol{k}$ integer $\geq$ **0**, the cosine of $\boldsymbol{\{n\pi/2 + (-1)^n(\pi/6)\}}$ is the same, and is the same as $\boldsymbol{\cos (\pi/3)}$, which is 1/2.

So if $\boldsymbol{n = 4k + 1}$ for $\boldsymbol{k}$ integer $\geq$ **0**, $\boldsymbol{\cos \{n\pi/2 + (-1)^n(\pi/6)\} = \cos \pi/3 = 1/2}$.

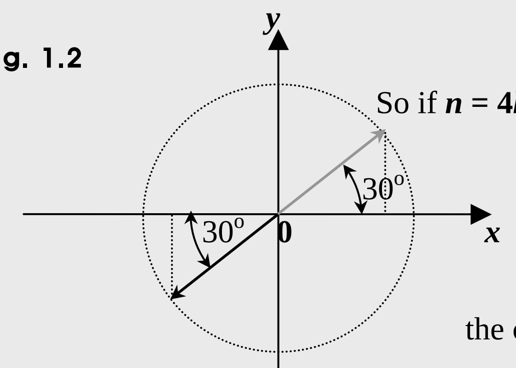
Next, we can notice that if $\boldsymbol{n = 4k + 2}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, all the vectors get the same spot again.

$\boldsymbol{n} = \mathbf{2} \Rightarrow \boldsymbol{n\pi/2 + (-1)^n(\pi/6) = 2\pi/2 + \pi/6 = \pi + \pi/6}$.

$\boldsymbol{n} = \mathbf{6} \Rightarrow \boldsymbol{n\pi/2 + (-1)^n(\pi/6) = 6\pi/2 + \pi/6 = 2\pi + \pi + \pi/6}$.

$\boldsymbol{n} = \mathbf{10} \Rightarrow \boldsymbol{n\pi/2 + (-1)^n(\pi/6) = 10\pi/2 + \pi/6 = 4\pi + \pi + \pi/6}$.

Fig. 1.2



So if $\boldsymbol{n = 4k + 2}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, all the vectors get the same spot in the third quadrant.

Thus, if $\boldsymbol{n = 4k + 2}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, the cosine of $\boldsymbol{\{n\pi/2 + (-1)^n(\pi/6)\}}$ is the same, and is the same as $\mathbf{cos}\ \boldsymbol{(\pi + \pi/6)}$, which is $-\frac{\sqrt{3}}{2}$.

That's because $\mathbf{cos}\ \boldsymbol{(\pi + \pi/6) = -}\mathbf{cos}\ \boldsymbol{\pi/6} = -\frac{\sqrt{3}}{2}$.

So if $\boldsymbol{n = 4k + 2}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, $\mathbf{cos}\ \boldsymbol{\{n\pi/2 + (-1)^n(\pi/6)\} = -}\mathbf{cos}\ \boldsymbol{\pi/6} = -\frac{\sqrt{3}}{2}$.

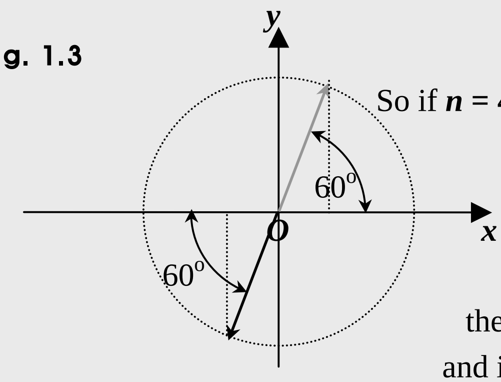
And next, we can notice that if $\boldsymbol{n = 4k + 3}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, all the vectors get the same spot again.

$\boldsymbol{n} = \mathbf{3} \Rightarrow \boldsymbol{n\pi/2 + (-1)^n(\pi/6) = 3\pi/2 - \pi/6 = \pi + \pi/2 - \pi/6 = \pi + \pi/3}$.

$\boldsymbol{n} = \mathbf{7} \Rightarrow \boldsymbol{n\pi/2 + (-1)^n(\pi/6) = 7\pi/2 - \pi/6 = 2\pi + 3\pi/2 - \pi/6 = 2\pi + \pi + \pi/3}$.

$\boldsymbol{n} = \mathbf{11} \Rightarrow \boldsymbol{n\pi/2 + (-1)^n(\pi/6) = 11\pi/2 - \pi/6 = 4\pi + 3\pi/2 - \pi/6 = 4\pi + \pi + \pi/3}$.

Fig. 1.3



So if $\boldsymbol{n = 4k + 3}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, all the vectors get the same spot in the third quadrant.

Thus, if $\boldsymbol{n = 4k + 3}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, the cosine of $\boldsymbol{\{n\pi/2 + (-1)^n(\pi/6)\}}$ is the same, and is the same as $\mathbf{\cos(\pi + \pi/3)}$, which is -1/2. That's because $\mathbf{\cos(\pi + \pi/3) = -\cos \pi/3 = -1/2}$.

So if $\boldsymbol{n = 4k + 3}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, $\boldsymbol{\cos\{n\pi/2 + (-1)^n(\pi/6)\} = -\cos \pi/3 = -1/2}$.

What if though, $\boldsymbol{k}$ is an integer negative?

Putting some negative values into $\boldsymbol{n}$ beginning with 0, we get a sequence below.

- $\boldsymbol{n = 0 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = \pi/6}$.

$\boldsymbol{n = -1 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -\pi/2 - \pi/6 = -2\pi/3}$.

$\boldsymbol{n = -2 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -2\pi/2 + \pi/6 = -\pi + \pi/6}$.

$\boldsymbol{n = -3 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -3\pi/2 - \pi/6 = -\pi - \pi/2 - \pi/6 = -\pi - 2\pi/3}$.

- $\boldsymbol{n = -4 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -4\pi/2 + \pi/6 = -2\pi + \pi/6}$.

$\boldsymbol{n = -5 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -5\pi/2 - \pi/6 = -2\pi - \pi/2 - \pi/6 = -2\pi - 2\pi/3}$.

$\boldsymbol{n = -6 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -6\pi/2 + \pi/6 = -2\pi - \pi + \pi/6}$.

$\boldsymbol{n = -7 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -7\pi/2 - \pi/6 = -2\pi - 3\pi/2 - \pi/6 = -2\pi - \pi - 2\pi/3}$.

- $\boldsymbol{n = -8 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -8\pi/2 + \pi/6 = -4\pi + \pi/6}$.

$n = -9 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -9\pi/2 - \pi/6 = -4\pi - \pi/2 - \pi/6 = -4\pi - 2\pi/3$.

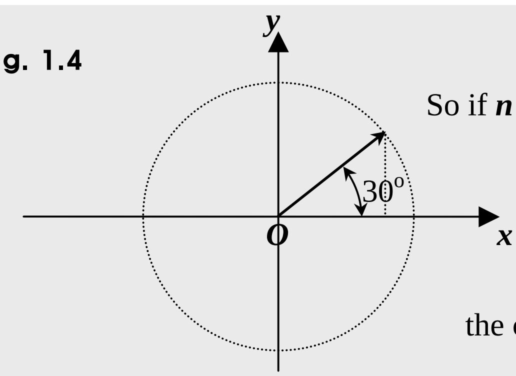
$n = -10 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -10\pi/2 + \pi/6 = -4\pi - \pi + \pi/6$.

$n = -11 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -11\pi/2 - \pi/6 = -4\pi + 3\pi/2 - \pi/6 = -4\pi - \pi - 2\pi/3$.

• $n = -12 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -12\pi/2 + \pi/6 = -6\pi + \pi/6$.

Now, taking a closer look at the sequence of angles above, we can notice again, if $n = 4k$ for k integer ≤ 0, all the vectors get the same spot again.

Fig. 1.4



So if $n = 4k$ for k integer ≤ 0, all the vectors get the same spot in the first quadrant.

Thus, if $n = 4k$ for k integer ≤ 0, the cosine of $\{n\pi/2 + (-1)^n(\pi/6)\}$ is the same, and is the same as $\cos \pi/6$, which is $\frac{\sqrt{3}}{2}$.

So if $n = 4k$ for k integer ≤ 0, $\cos \{n\pi/2 + (-1)^n(\pi/6)\} = \frac{\sqrt{3}}{2}$.

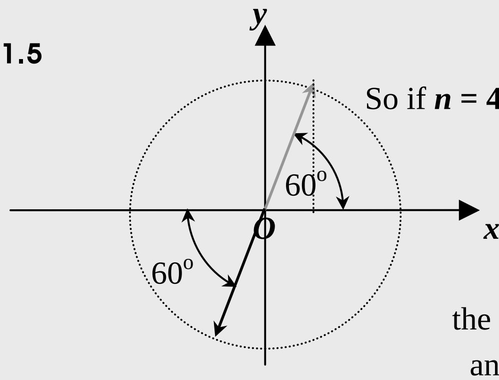
Next, we can notice that if $n = 4k - 1$ for k integer ≤ 0, all the vectors get the same spot again.

$n = -1 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -\pi/2 - \pi/6 = -2\pi/3$.

$n = -5 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -5\pi/2 - \pi/6 = -2\pi - \pi/2 - \pi/6 = -2\pi - 2\pi/3$.

$n = -9 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -9\pi/2 - \pi/6 = -4\pi - \pi/2 - \pi/6 = -4\pi - 2\pi/3$.

Fig. 1.5



So if $\boldsymbol{n} = \mathbf{4}\boldsymbol{k} - \mathbf{1}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, all the vectors get the same spot in the third quadrant.

Thus, if $\boldsymbol{n} = \mathbf{4}\boldsymbol{k} - \mathbf{1}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, the cosine of $\{n\pi/2 + (-1)^n(\pi/6)\}$ is the same, and is the same as $-\cos(\pi/3)$, which is -1/2.

So if $\boldsymbol{n} = \mathbf{4}\boldsymbol{k} - \mathbf{1}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, $\cos\{n\pi/2 + (-1)^n(\pi/6)\} = -\cos \pi/3 = -1/2$.

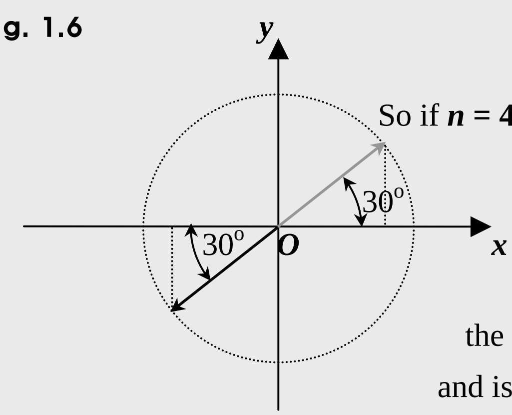
Next, we can notice that if $\boldsymbol{n} = \mathbf{4}\boldsymbol{k} - \mathbf{2}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, all the vectors get the same spot again.

$n = -2 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -2\pi/2 + \pi/6 = -\pi + \pi/6$.

$n = -6 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -6\pi/2 + \pi/6 = -2\pi - \pi + \pi/6$.

$n = -10 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -10\pi/2 + \pi/6 = -4\pi - \pi + \pi/6$.

Fig. 1.6



So if $\boldsymbol{n} = \mathbf{4}\boldsymbol{k} - \mathbf{2}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, all the vectors get the same spot in the third quadrant.

Thus, if $\boldsymbol{n} = \mathbf{4}\boldsymbol{k} - \mathbf{2}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, the cosine of $\{n\pi/2 + (-1)^n(\pi/6)\}$ is the same, and is the same as $\cos(-\pi + \pi/6)$, which is $-\frac{\sqrt{3}}{2}$.

That's because $\cos(-\pi + \pi/6) = -\cos \pi/6 = -\frac{\sqrt{3}}{2}$.

So if $\boldsymbol{n} = \mathbf{4}\boldsymbol{k} - \mathbf{2}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, $\cos\{n\pi/2 + (-1)^n(\pi/6)\} = -\cos \pi/6 = -\frac{\sqrt{3}}{2}$.

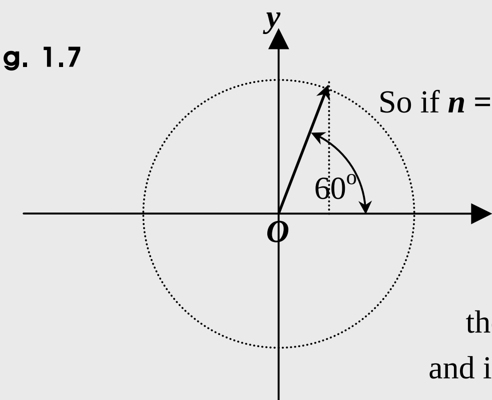
And next, we can notice that if $\boldsymbol{n = 4k - 3}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, all the vectors get the same spot again.

$\boldsymbol{n = -3 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -3\pi/2 - \pi/6 = -\pi - \pi/2 - \pi/6 = -\pi - 2\pi/3}$.

$\boldsymbol{n = -7 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -7\pi/2 - \pi/6 = -2\pi - 3\pi/2 - \pi/6 = -2\pi - \pi - 2\pi/3}$.

$\boldsymbol{n = -11 \Rightarrow n\pi/2 + (-1)^n(\pi/6) = -11\pi/2 - \pi/6 = -4\pi + 3\pi/2 - \pi/6 = -4\pi - \pi - 2\pi/3}$.

Fig. 1.7



So if $\boldsymbol{n = 4k - 3}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, all the vectors get the same spot in the first quadrant.

And thus, if $\boldsymbol{n = 4k - 3}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, the cosine of $\boldsymbol{\{n\pi/2 + (-1)^n(\pi/6)\}}$ is the same, and is the same as $\mathbf{cos\ (-\pi - 2\pi/3)}$, which is 1/2. That's because $\mathbf{cos\ (-\pi - 2\pi/3) = cos\ \pi/3 = 1/2}$.

Thus, if $\boldsymbol{n = 4k - 3}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, $\mathbf{cos}\ \boldsymbol{\{n\pi/2 + (-1)^n(\pi/6)\} = \mathbf{cos}\ \pi/3 = 1/2}$.

So putting together now, the case where $\boldsymbol{k}$ is greater than or equal to 0, and the case where $\boldsymbol{k}$ is less than 0, we can put them the way below.

If $\boldsymbol{n = 4k}$ for $\boldsymbol{k}$ integer, $\mathbf{cos}\ \boldsymbol{\{n\pi/2 + (-1)^n(\pi/6)\}} = \frac{\sqrt{3}}{2}$.

If $\boldsymbol{n = 4k + 1}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, $\mathbf{cos}\ \boldsymbol{\{n\pi/2 + (-1)^n(\pi/6)\} = 1/2}$.

If $\boldsymbol{n = 4k - 3}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, $\mathbf{cos}\ \boldsymbol{\{n\pi/2 + (-1)^n(\pi/6)\} = 1/2}$.

If $\boldsymbol{n = 4k + 3}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, $\mathbf{cos}\ \boldsymbol{\{n\pi/2 + (-1)^n(\pi/6)\} = -1/2}$.

If $\boldsymbol{n = 4k - 1}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, $\mathbf{cos}\ \boldsymbol{\{n\pi/2 + (-1)^n(\pi/6)\} = -1/2}$.

If $\boldsymbol{n = 4k + 2}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, $\mathbf{cos}\ \boldsymbol{\{n\pi/2 + (-1)^n(\pi/6)\}} = -\frac{\sqrt{3}}{2}$.

And if $\boldsymbol{n = 4k - 2}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, $\mathbf{cos}\ \boldsymbol{\{n\pi/2 + (-1)^n(\pi/6)\}} = -\frac{\sqrt{3}}{2}$.

However, as in the case of the sine in the Problem 0 in the Example 2, putting together for instance, the case where $\boldsymbol{n = 4k + 1}$ for $\boldsymbol{k}$ integer $\geq \mathbf{0}$, and the case where $\boldsymbol{n = 4k - 3}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, we can just put both together the way as follows.

$$\boldsymbol{n = 4k + 1} \text{ for } \boldsymbol{k} \text{ integer}$$

And the same is true, too, for the other cases where we get the same ratios.
So we can put together all the cases above the way below.

If $\boldsymbol{n = 4k}$ for $\boldsymbol{k}$ integer, $\mathbf{cos}\ \{n\pi/2 + (-1)^n(\pi/6)\} = \frac{\sqrt{3}}{2}$.

If $\boldsymbol{n = 4k + 1}$ for $\boldsymbol{k}$ integer, $\mathbf{cos}\ \{n\pi/2 + (-1)^n(\pi/6)\} = 1/2$.

If $\boldsymbol{n = 4k + 2}$ for $\boldsymbol{k}$ integer, $\mathbf{cos}\ \{n\pi/2 + (-1)^n(\pi/6)\} = -\frac{\sqrt{3}}{2}$.

And if $\boldsymbol{n = 4k + 3}$ for $\boldsymbol{k}$ integer, $\mathbf{cos}\ \{n\pi/2 + (-1)^n(\pi/6)\} = -1/2$.

And of course, we can put together all the cases above the way below, too.

If $\boldsymbol{n = 4k}$ for $\boldsymbol{k}$ integer, $\mathbf{cos}\ \{n\pi/2 + (-1)^n(\pi/6)\} = \frac{\sqrt{3}}{2}$.

If $\boldsymbol{n = 4k - 1}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, $\mathbf{cos}\ \{n\pi/2 + (-1)^n(\pi/6)\} = -1/2$.

If $\boldsymbol{n = 4k - 2}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, $\mathbf{cos}\ \{n\pi/2 + (-1)^n(\pi/6)\} = -\frac{\sqrt{3}}{2}$.

If $\boldsymbol{n = 4k - 3}$ for $\boldsymbol{k}$ integer $\leq \mathbf{0}$, $\mathbf{cos}\ \{n\pi/2 + (-1)^n(\pi/6)\} = 1/2$.

So putting threads together, we want to consider the four cases
where $\boldsymbol{n = 4k}$, $\boldsymbol{4k + 1}$, $\boldsymbol{4k + 2}$, and $\boldsymbol{4k + 3}$ for $\boldsymbol{k}$ is an integer.