Examples in Sine Functions

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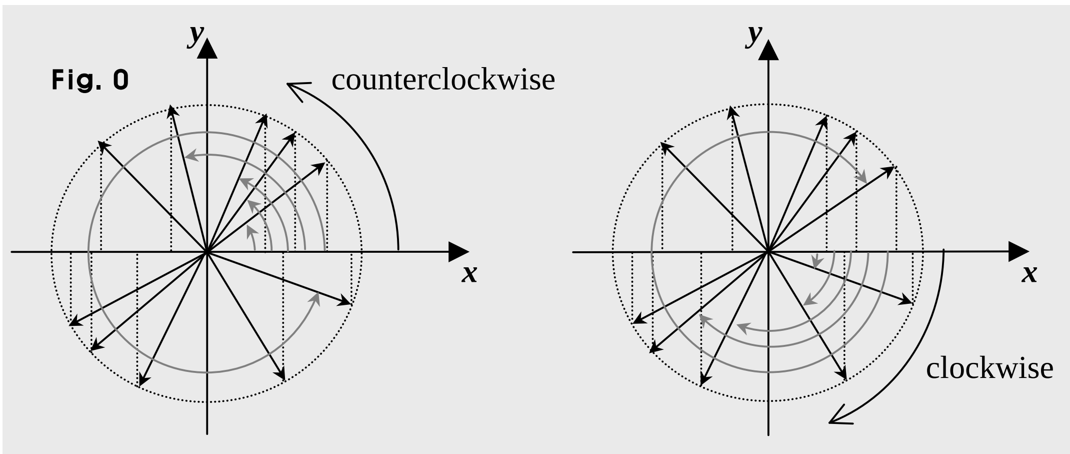


In coordinate trigonometry, the vector turning makes the angles. If turning clockwise, it makes angles negative, and if counterclockwise, it makes angles positive. And of course, if no turning, the angle made is 0.

So the angle can be 0^o or any angle positive or negative.
In each right triangle, the vector is of length 1, and is the hypotenuse.
And (x, y) is the terminal point, so x is the adjacent, and y is the opposite.

Thus, assuming θ is the angle, we get $\sin\theta = y$.
So if the vector is in the first quadrant, $\sin\theta > 0$, since $y > 0$.
In the second, $\sin\theta > 0$, too, since $y > 0$.
In the third, $\sin\theta < 0$, because $y < 0$. And in the fourth, $\sin\theta < 0$, too, since $y < 0$.

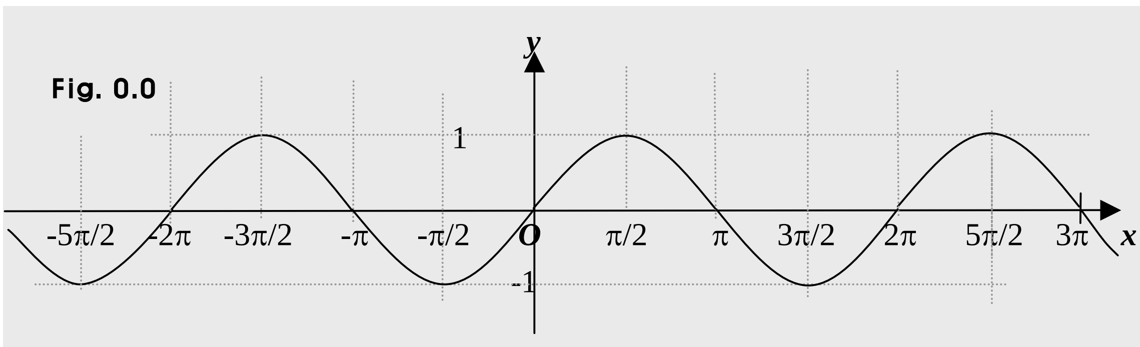
Put in a graph the curve of each of the equations below.

0. $y = \sin|2x|$
1. $|y| = \sin 2x$
2. $|y| = \sin|2x|$
3. $y = \sin x + \sin|x|$

Suggestions or Solutions
To the Problem in the Example 0

Put in a graph the equation as follows: $y = \sin|2x|$.

To begin with, putting in the x-y plane, the curve of the sine function $y = f(x) = \sin x$, we can put it the way below.

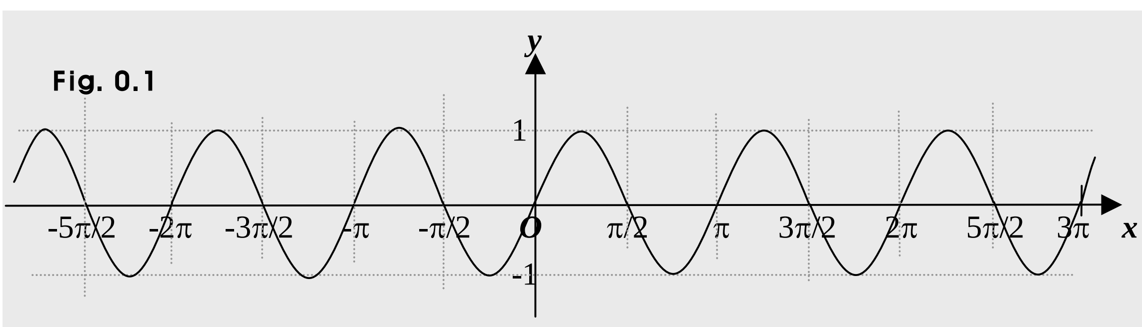


The curve above is in fact, the same as the curve of an equation $y = \sin x$.

And the curve of an equation $y = \sin 2x$ is the curve of a function $y = g(x) = \sin 2x$, too.

We know in the curve of the function g, the frequency is 2, so the period is $2\pi/2 = \pi$.

So putting in a graph, the curve of the equation $y = \sin 2x$, we can put it this way:



What then, about the curve of $y = \sin|x|$?

In that case, we want to consider two cases, one is $x \geq 0$, and the other is $x < 0$. Why?

Though looking awkward, $\sin|x|$ can be put this way, too: $\sin t$ where $t = |x|$.

And we know $|x| \geq 0$ for all x.

So even if $x < 0$, that is, x gets a negative value, we still get $t > 0$, that is, t gets a positive value.

Thus, for instance, even if $x = -\pi/6$, we need to get $\sin \pi/6$, because $|-\pi/6| = \pi/6$.

So given $y = \sin |x|$ for $x \geq 0$, we can just set $y = \sin x$.

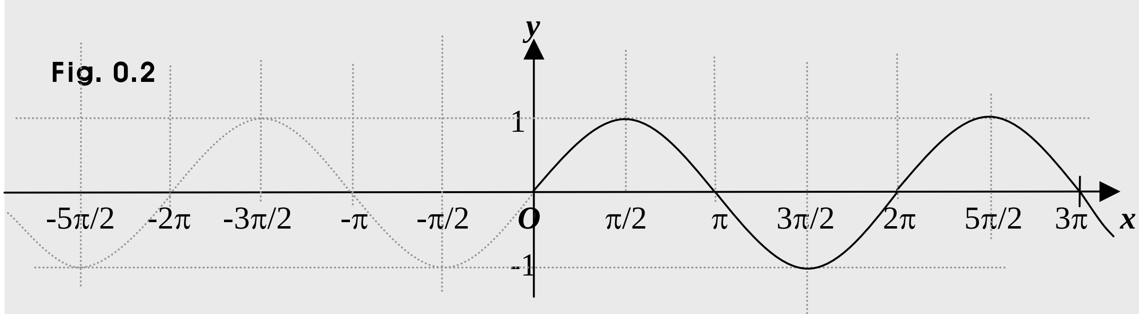
If however, $x < 0$, we want to set $y = \sin(-x)$, because $-x > 0$, since $|x| \geq 0$ in $\sin |x|$.

And we have a trig identity $\sin(-x) = -\sin x$.

Thus, given $y = \sin |x|$, we get $y = \sin x$ for $x \geq 0$, and $y = -\sin x$ for $x < 0$.

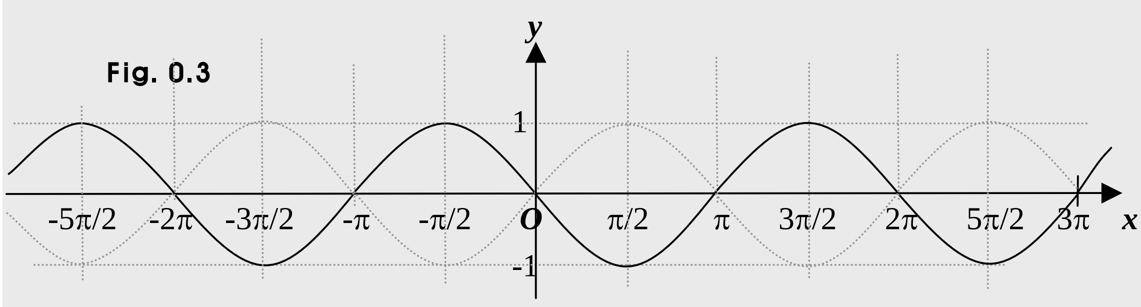
So taking care of first, the curve of $y = \sin x$ for $x \geq 0$, we can put it the way below.

Fig. 0.2



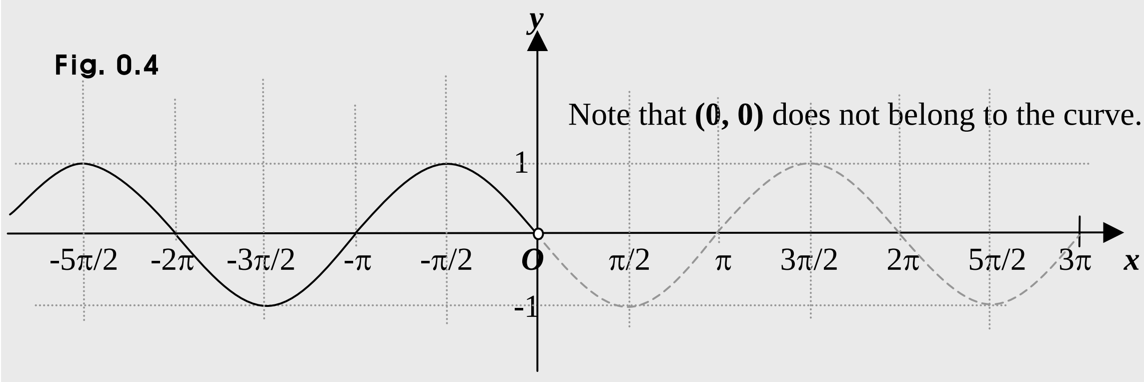
Next, moving on to the curve of $y = -\sin x$ for x real, we can put it the way below.

Fig. 0.3



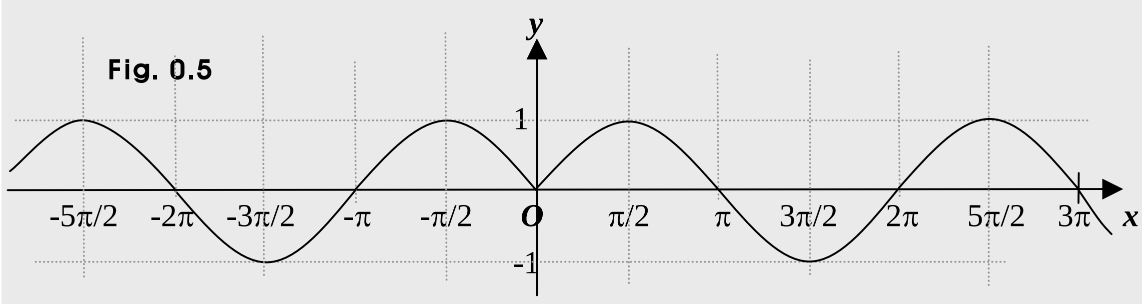
So next, moving on to the curve of $y = -\sin x$ for $x < 0$, we can put it the way below.

Fig. 0.4



Thus, putting in a graph, the curve of $\boldsymbol{y = \sin |x|}$, we can put it the way as follows.

Fig. 0.5



What then about the curve of the equation $\boldsymbol{y = \sin |2x|}$?

We know $\boldsymbol{|2x| \geq 0}$ for all $\boldsymbol{x}$. So we want to consider two cases, $\boldsymbol{x \geq 0}$, and $\boldsymbol{x < 0}$.

That is, $\boldsymbol{y = \sin |2x|}$ is the same as $\boldsymbol{y = \sin 2|x|}$.

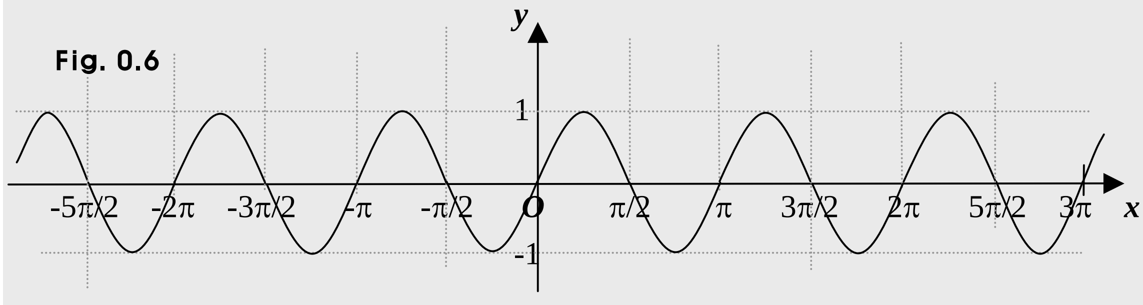
So given $\boldsymbol{y = \sin 2|x|}$, and if $\boldsymbol{x \geq 0}$, we can set $\boldsymbol{y = \sin 2x}$.

If however, $\boldsymbol{x < 0}$, we want to set $\boldsymbol{y = \sin (-2x)}$ because $\boldsymbol{-2x > 0}$.

Thus, we get $\boldsymbol{y = \sin 2x}$ for $\boldsymbol{x \geq 0}$, and we get $\boldsymbol{y = -\sin 2x}$ for $\boldsymbol{x < 0}$.

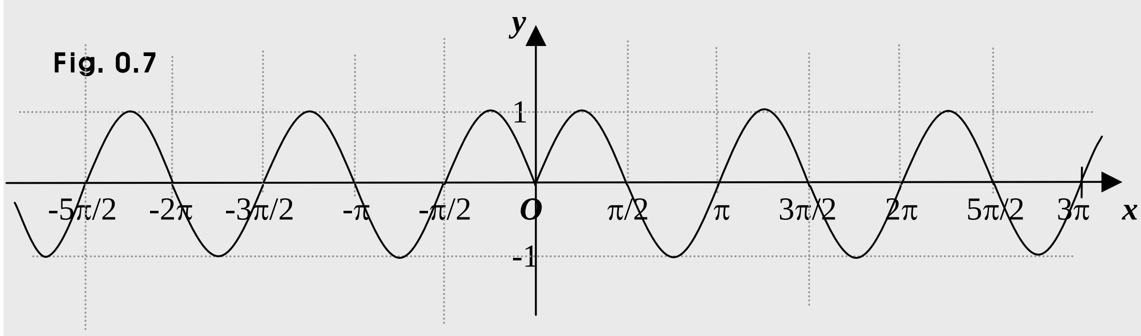
And we can put in a graph, the curve of $\boldsymbol{y = \sin 2x}$ the way below.

Fig. 0.6



So we can put in a graph, the curve of the equation ***y* = sin |2*x*|** the way below.

Fig. 0.7



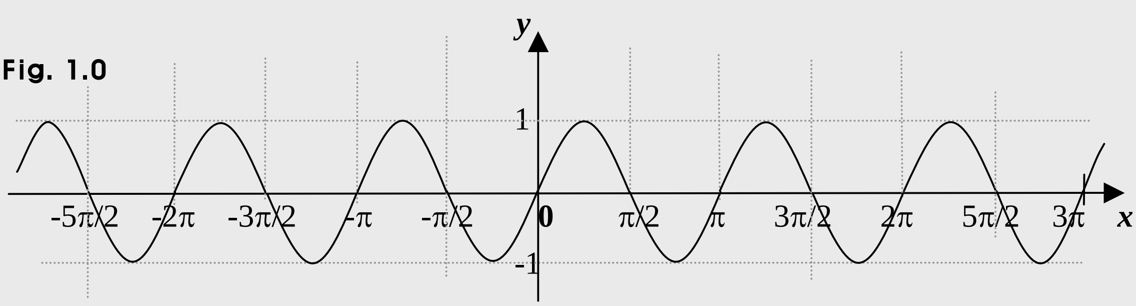
Notice that the curve is symmetric about the ***y***-axis.

Suggestions or Solutions
To the Problem in the Example 1

Put in a graph the equation as follows: $|y| = \sin 2x$.

To begin with, putting in the x-y plane, the curve of the equation $y = \sin 2x$, we can put it the way below.

Fig. 1.0



What then, about the curve of $|y| = \sin 2x$?

In that case, we want to consider two cases, one is $y \geq 0$, and the other is $y < 0$.   Why?

Though looking awkward, $|y| = \sin 2x$ can be put this way, too: $t = \sin 2x$ where $t = |y|$.

And we know $|y| \geq 0$ for all y.

So even if $y < 0$, that is, y gets a negative value, we still get $t > 0$, that is, t gets a positive value.
So for instance, even if $y = -1/2$, we need to get $t = 1/2$, because $|-1/2| = 1/2$.

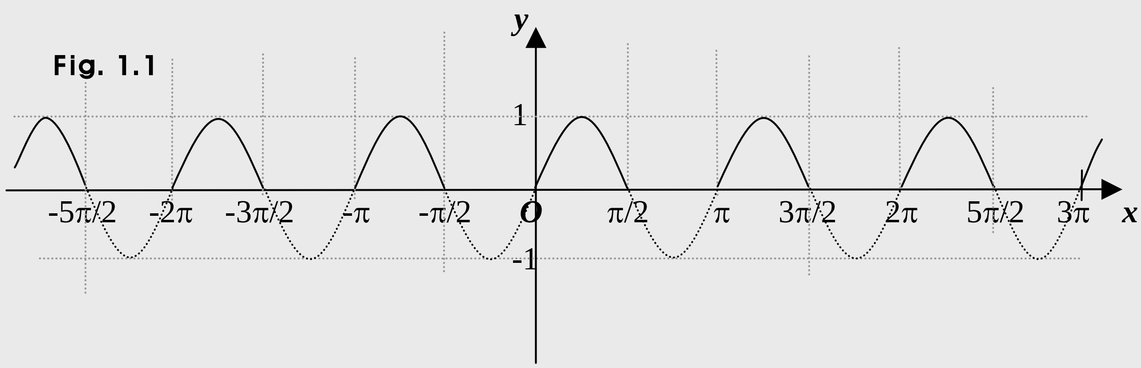
So given $|y| = \sin 2x$ for $y \geq 0$, we can just set $y = \sin 2x$.

If however, $y < 0$, we want to set $-y = \sin 2x$ because $-y > 0$, since $|y| \geq 0$ in $|y| = \sin 2x$.

Thus, we get $y = \sin 2x$ for $y \geq 0$, and we get $y = -\sin 2x$ for $y < 0$.

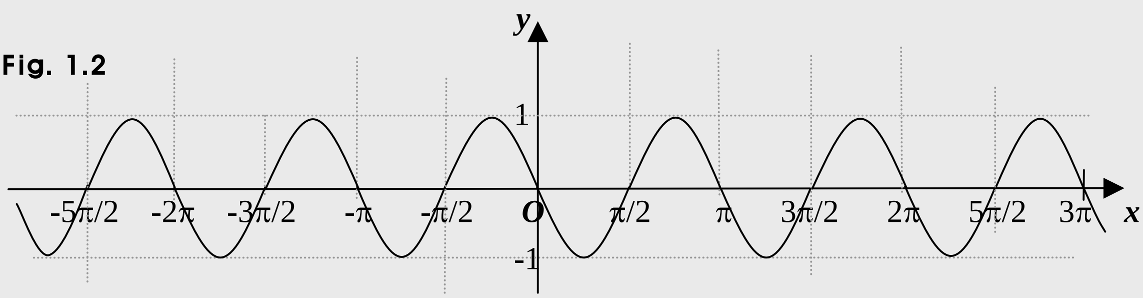
And taking care of first, the curve of $y = \sin 2x$ for $y \geq 0$, we can put it this way:

Fig. 1.1



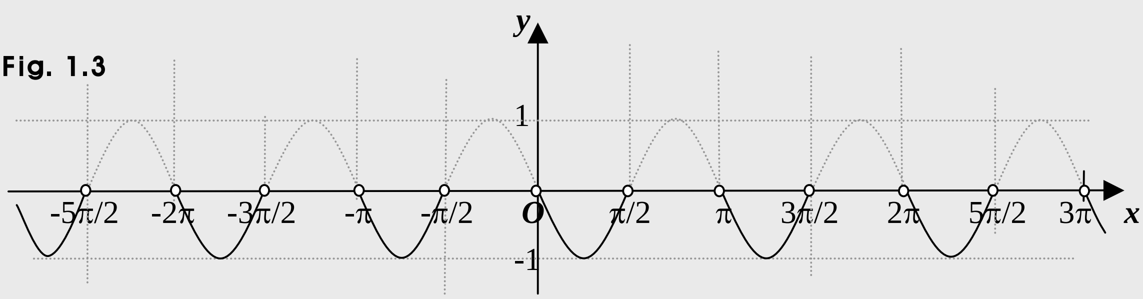
Next, we can put in a graph, the curve of $y = -\sin 2x$ the way below:
(Note that since no domain is specified, "$y = -\sin 2x$" implies "$y = -\sin 2x$ for x real.")

Fig. 1.2



So next, moving on to the curve of $y = -\sin 2x$ for $y < 0$, we can put it this way:

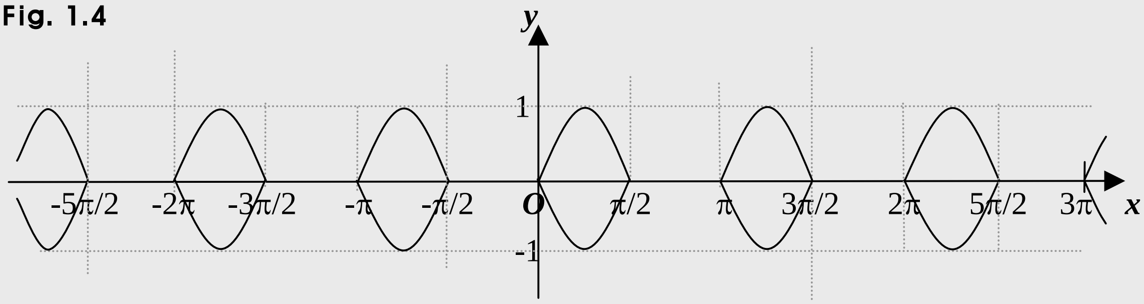
Fig. 1.3



Note that all the points $(n\pi/2, 0)$ for n integer, do not belong to the curve above.

Thus, putting in a graph, the curve of $|y| = \sin 2x$, we can put it the way as follows.

Fig. 1.4



Notice that the curve is symmetric about the ***x***-axis.

In fact, the curve of an equation $|y| = f(x)$ is symmetric about the ***x***-axis.

For instance, assuming $f(x) = x^2 - 3x + 2$, we can get $|y| = x^2 - 3x + 2$, and the curve of it is symmetric about the ***x***-axis.

And assuming $f(x) = \sin x + 2x + 1$, we can get $|y| = \sin x + 2x + 1$, and the curve of it is symmetric about the ***x***-axis, too.
What then about the curve of $y = f(|x|)$?

It is symmetric about the ***y***-axis,
IN the example 0 above, we have $y = \sin |2x|$, which is equal to $y = \sin 2|x|$, since we have $|2x| = 2|x|$.
So assuming $y = f(x) = y = \sin 2x$, we get $y = f(|x|) = y = \sin 2|x|$,
which is equal to $y = \sin |2x|$,
Thus, the curve of an equation $y = f(|x|)$ is symmetric about the ***y***-axis.

In short,
the curve of $|y| = f(x)$ is symmetric about the ***x***-axis.
the curve of $y = f(|x|)$ is symmetric about the ***y***-axis.

Suggestions or Solutions
To the Problem in the Example 2

Put in a graph the equation as follows: $|y| = \sin |2x|$.

To begin with, putting in the x-y plane, the curve of the equation $y = \sin 2x$, we can put it the way below.

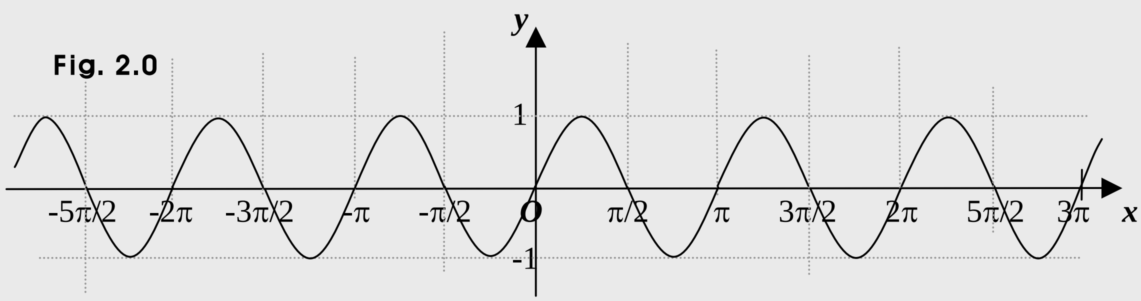


Fig. 2.0

Next, we know $|y| \geq 0$ for all y, and $|2x| \geq 0$ for all x.

So we want to consider four different cases as follows.

First, assuming $x \geq 0$, and $y \geq 0$, we get $y = \sin 2x$.

Next, assuming $x \geq 0$, and $y < 0$, we get $-y = \sin 2x$, so we get $y = -\sin 2x$.

Next, assuming $x < 0$, and $y < 0$, we get $-y = \sin(-2x)$, so we get $y = \sin 2x$.

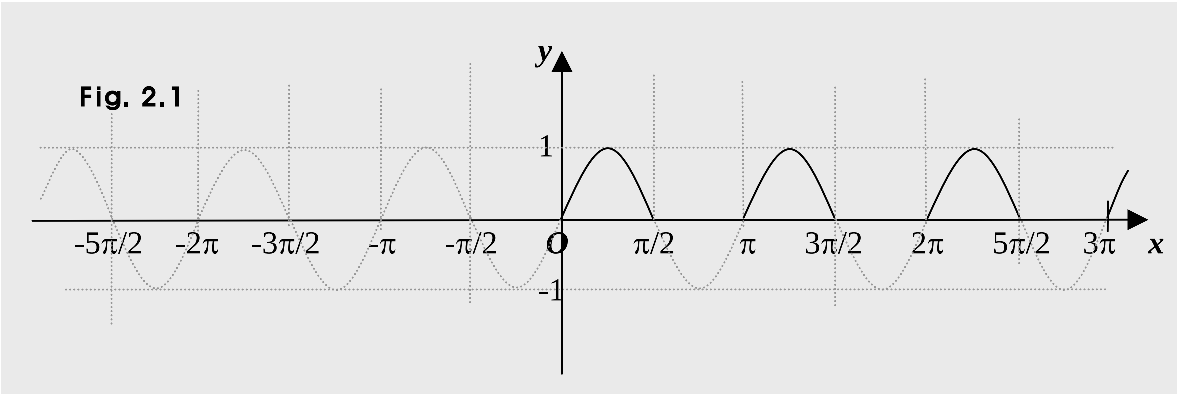
And next, assuming $x < 0$, and $y \geq 0$, we get $y = \sin(-2x)$, so we get $y = -\sin 2x$.

Thus in sum, we have these:

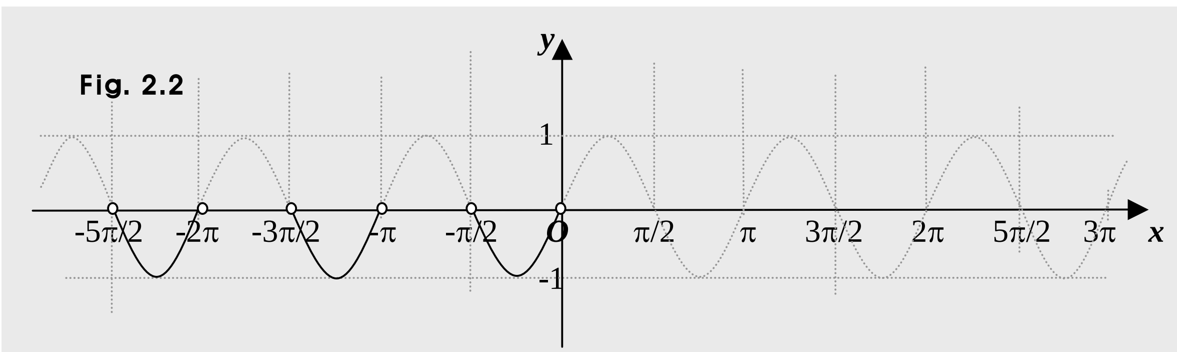
$y = \sin 2x$ for $x \geq 0$, and $y \geq 0$. $y = \sin 2x$ for $x < 0$, and $y < 0$.

$y = -\sin 2x$ for $x \geq 0$, and $y < 0$. $y = -\sin 2x$ for $x < 0$, and $y \geq 0$.

So beginning with the curve of $y = \sin 2x$ for $x \geq 0$, and $y \geq 0$, we can get this:

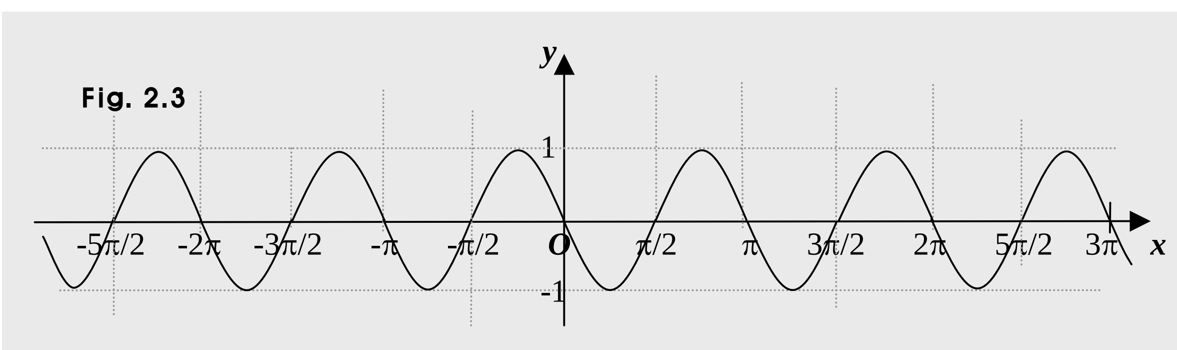


Next, moving on to the curve of $\boldsymbol{y = \sin 2x}$ for $\boldsymbol{x < 0}$, and $\boldsymbol{y < 0}$, we can get this:



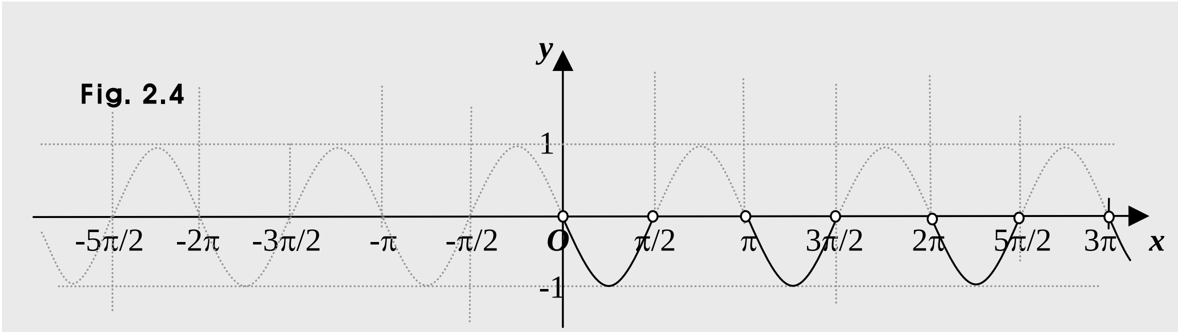
Note that all the points $\boldsymbol{(n\pi/2, 0)}$ for $\boldsymbol{n}$ integer $\leq \boldsymbol{0}$, do not belong to the curve above.

Next, we can put in a graph, the curve of $\boldsymbol{y = -\sin 2x}$ the way below.



So next, moving on to the curve of $\boldsymbol{y = -\sin 2x}$ for $\boldsymbol{x \geq 0}$, and $\boldsymbol{y < 0}$, we get this:

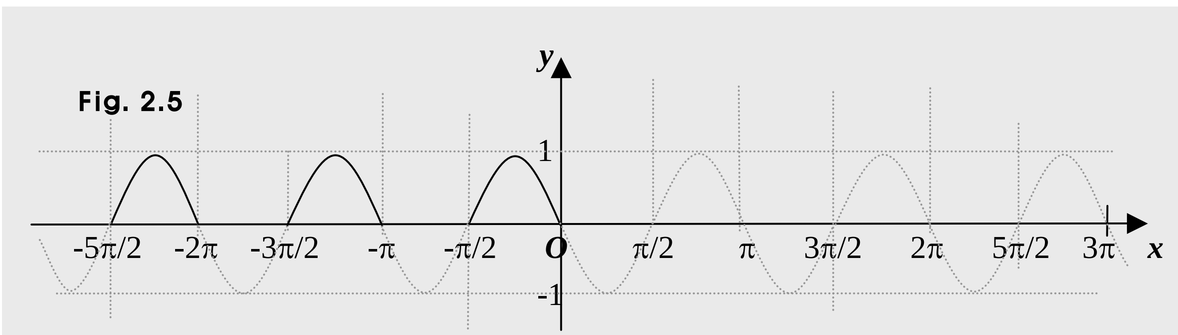
Fig. 2.4



Note that all the points **(nπ/2, 0)** for ***n*** integer ≥ **0**, do not belong to the curve above.

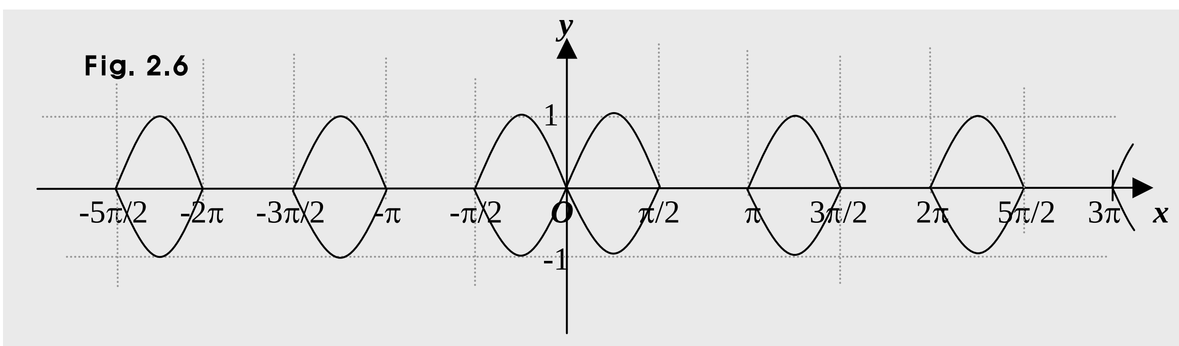
And next, moving on to the curve of ***y* = –sin 2*x*** for ***x* < 0**, and ***y* ≥ 0**, we can get this:

Fig. 2.5



Thus, putting in a graph, the curve of **|*y*| = sin |2*x*|**, we can put it the way as follows.

Fig. 2.6



Notice that the curve is symmetric about the origin, as well as the ***x***-axis and the ***y***-axis. In fact, the curve of an equation **|*y*| = *f*(|*x*|)** is

> symmetric about the origin, as well as the ***x***-axis and the ***y***-axis.

For instance, assuming ***f*(|*x*|) = sin |*x*| + 2|*x*| + 1**, we can get **|*y*| = sin |*x*| + 2|*x*| + 1**,, and the curve of it is symmetric about the origin, as well as the ***x***-axis and the ***y***-axis.

Suggestions or Solutions
To the Problem in the Example 3

Put in a graph the equation as follows: *y* = sin *x* + sin |*x*|.

To begin with, we can put in the ***x-y*** plane, the curve of ***y* = sin *x***, the way below.

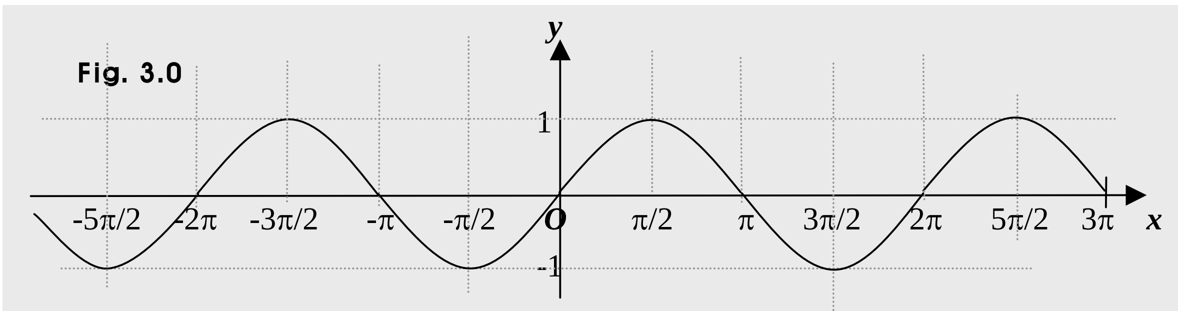


Fig. 3.0

And next, putting in a graph, the curve of ***y* = sin |*x*|**, we can put it the way below.

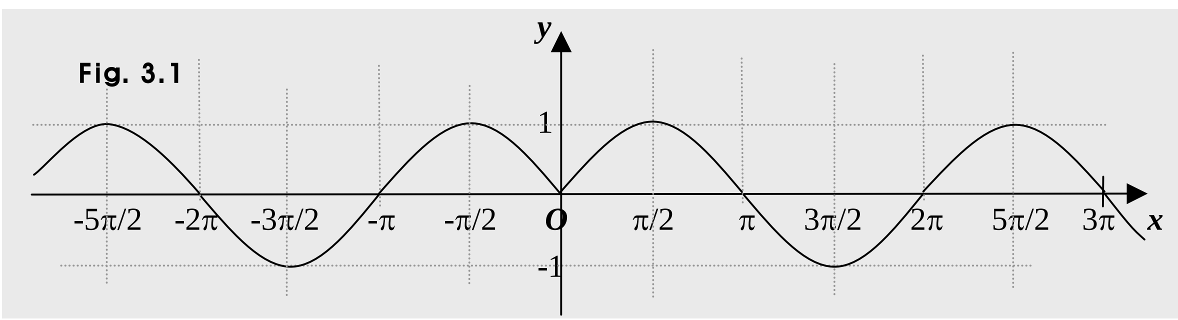


Fig. 3.1

So adding together the two curves above, we can get this:

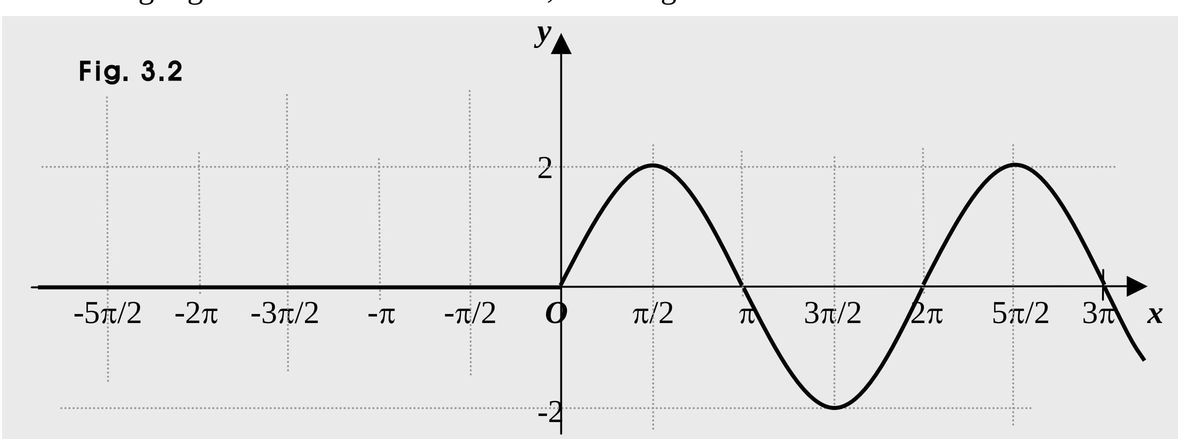


Fig. 3.2

Thus, the curve above is the curve of the equation ***y* = sin *x* + sin |*x*|**.

Let's take a look at now, how we can get the curve algebraically.

To begin with, we know $|x| \geq 0$ for all x.
So we want to put the equation in two cases.

One is a case where $x \geq 0$, and the other is a case where $x < 0$.
Thus, removing the absolute sign, we can get these:

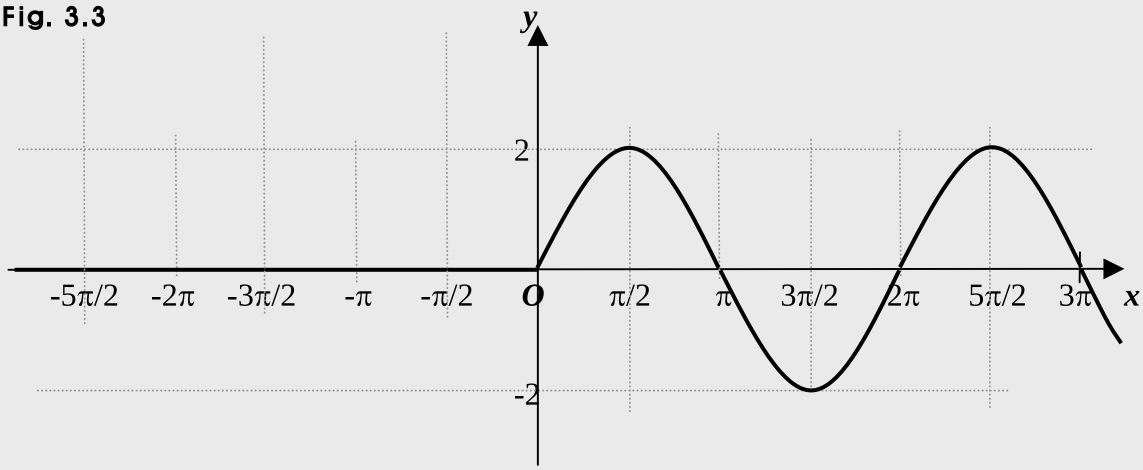
If $x \geq 0$, $y = \sin x + \sin |x| = \sin x + \sin x = 2\sin x$.

And if $x < 0$, $y = \sin x + \sin |x| = \sin x + \sin(-x) = \sin x - \sin x = 0$.

So we get $y = 2\sin x$ for $x \geq 0$, and $y = 0$ for $x < 0$.

Thus, we can put in a graph, the curve of $y = \sin x + \sin |x|$ the way below.

Fig. 3.3



And we can put the idea above the way below, too.

Assuming $f(x) = \sin x$, and $g(x) = \sin |x|$, we can set $h(x) = f(x) + g(x)$.
And the curve of the function h is the same as that of the equation $y = \sin x + \sin |x|$.

So for each value of x, adding together the values of $f(x)$ and $g(x)$, we get the value of $h(x)$, which is in turn, the value of y in the equation $y = \sin x + \sin |x|$.

For instance, we get $\boldsymbol{h(1) = f(1) + g(1)}$ for $\boldsymbol{x = 1}$.

And the $\boldsymbol{y}$-value for $\boldsymbol{x = 1}$ is $\mathbf{\sin 1 + \sin |1| = \sin 1 + \sin 1 = 2\sin 1}$.

Also, we get $\boldsymbol{h(-1) = f(-1) + g(-1)}$ for $\boldsymbol{x = -1}$.

And the $\boldsymbol{y}$-value for $\boldsymbol{x = -1}$ is $\mathbf{\sin (-1) + \sin |-1| = 0}$, because $\mathbf{\sin (-1) = -\sin 1}$.

So we can see that we get $\boldsymbol{y = 2\sin x}$ for $\boldsymbol{x \geq 0}$, and $\boldsymbol{y = 0}$ for $\boldsymbol{x < 0}$.

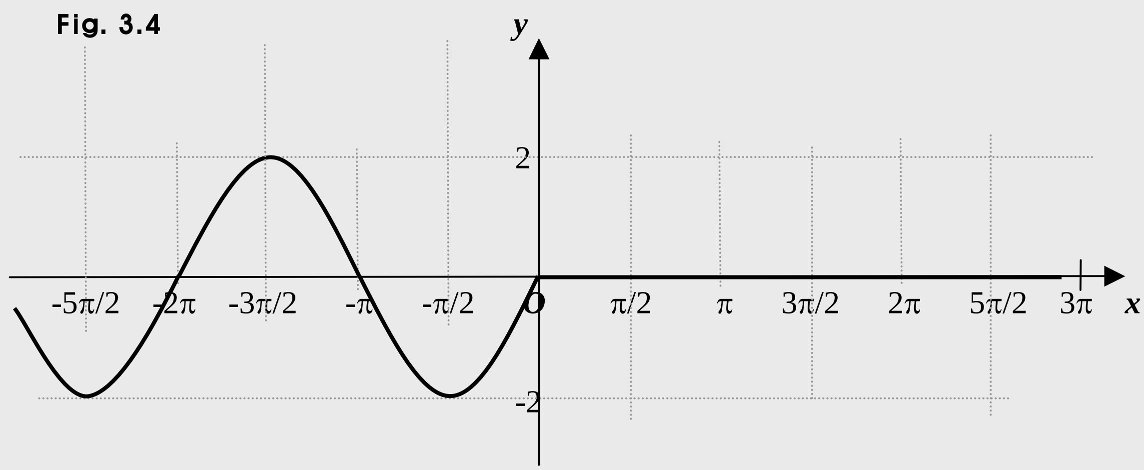
What then about the curve of $\boldsymbol{y = \sin x - \sin |x|}$?

If $\boldsymbol{x \geq 0}$, $\boldsymbol{y = \sin x - \sin |x| = \sin x - \sin x = 0}$.

And if $\boldsymbol{x < 0}$, $\boldsymbol{y = \sin x - \sin |x| = \sin x - \sin (-x) = \sin x + \sin x = 2\sin x}$.

Thus, we can put in a graph, the curve of $\boldsymbol{y = \sin x - \sin |x|}$ the way below.

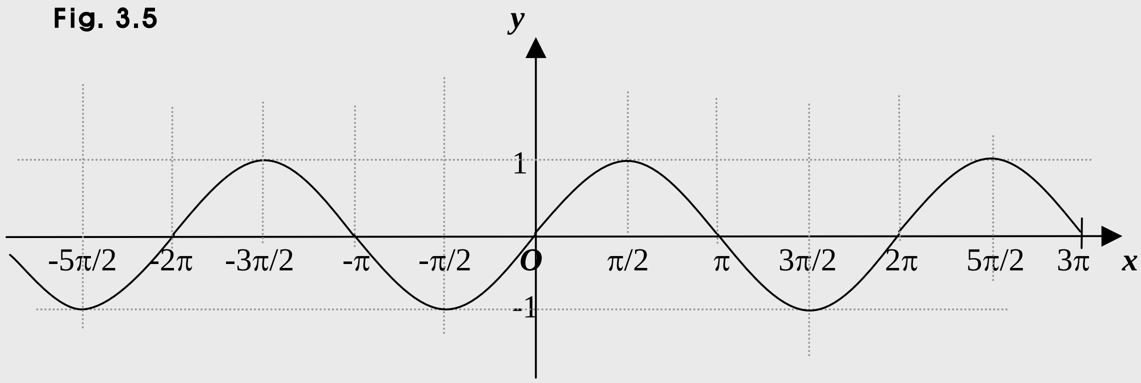
Fig. 3.4



What then about the curve of $\boldsymbol{y = \sin^2 x}$?

First, the curve of $\boldsymbol{y = \sin x}$ is as follows.

Fig. 3.5



So we can expect the curve of $\boldsymbol{y = \sin^2 x}$ to be made the way as follows.

Fig. 3.6