Examples in Cosine Functions

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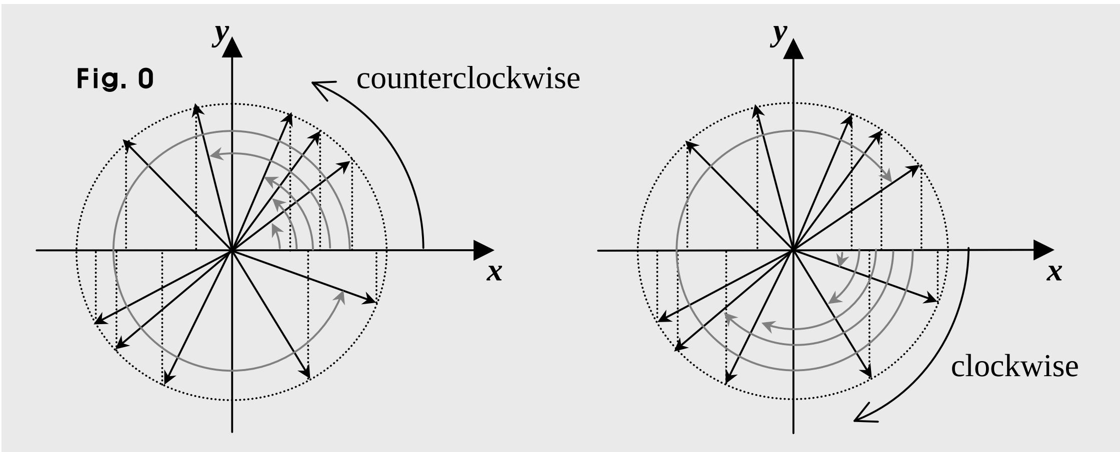


In coordinate trigonometry, the vector turning makes the angles. If turning clockwise, it makes angles negative, and if counterclockwise, it makes angles positive.

And of course, if no turning, the angle made is 0. So the angle can be 0^o or any angle positive or negative. In each right triangle, the vector is of length 1, and is the hypotenuse. And (x, y) is the terminal point, so x is the adjacent, and y is the opposite.

Thus, assuming θ is the angle, we get $\cos\theta = x$.

So if the vector is in the first quadrant, $\cos\theta > 0$, since $x > 0$.
In the second, $\cos\theta < 0$, since $x < 0$.
In the third, $\cos\theta < 0$, too, because $x < 0$. And in the fourth, $\cos\theta > 0$, since $x > 0$.

Put in a graph the curve of each of the equations below.

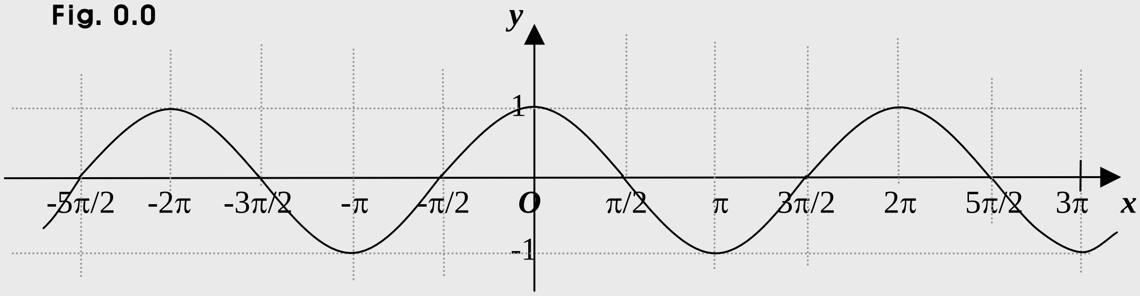
0. $y = \cos|2x|$
1. $|y| = \cos 2x$
2. $|y| = \cos|2x|$
3. $y = \cos x + \cos|x|$
4. $y = \cos x + \sin x$

Suggestions or Solutions
To the Problem in the Example 0

Put in a graph the equation as follows: $y = \cos |2x|$.

To begin with, putting in the x-y plane, the curve of the cosine function $y = f(x) = \cos x$, we can put it the way below.

Fig. 0.0



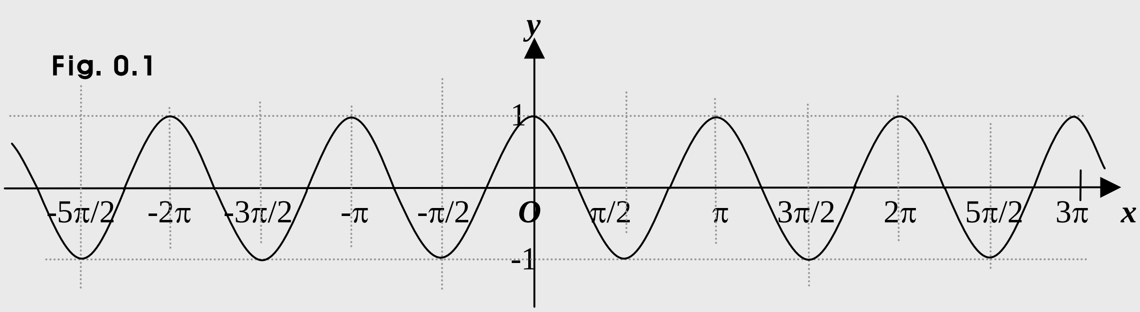
The curve above is in fact, the same as the curve of an equation $y = \cos x$.

And the curve of an equation $y = \cos 2x$ is the curve of a function $y = g(x) = \cos 2x$, too.

We know in the curve of the function g, the frequency is 2, so the period is $2\pi/2 = \pi$.

So putting in a graph, the curve of the equation $y = \cos 2x$, we can put it the way below.

Fig. 0.1



What then about the curve of $y = \cos |x|$?

It is the same as the curve of the equation $y = \cos x$.   How?

Though looking awkward, **cos** $|x|$ can be put this way, too: **cos** t where $t = |x|$.

And we know $|x| \geq 0$ for all x.
So even if $x < 0$, that is, x gets a negative value, we still get $t > 0$, that is, t gets a positive value.

Thus, for instance, even if $x = -\pi/3$, we need to get $\cos \pi/3$, because $|-\pi/3| = \pi/3$.

So given $y = \cos |x|$ for $x \geq 0$, we can just set $y = \cos x$.

If however, $x < 0$, we want to set $y = \cos (-x)$ because $-x > 0$, since $|x| \geq 0$ in $\cos |x|$.

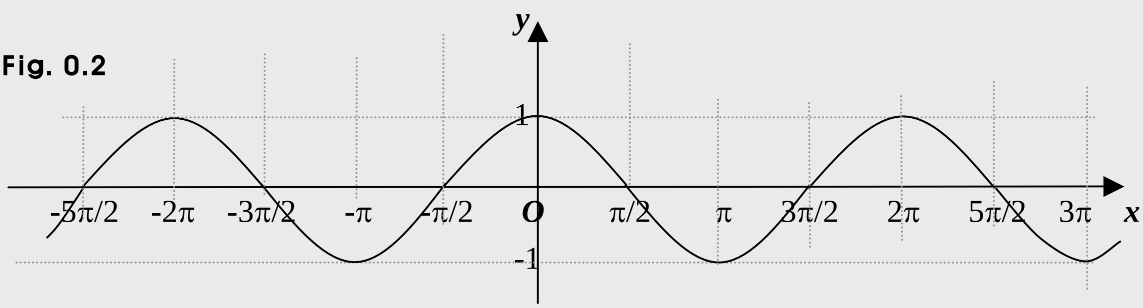
We have a trig identity though, which is $\cos (-x) = \cos x$.

So given $y = \cos |x|$, we get $y = \cos x$ for $x \geq 0$, and for $x < 0$, too.

Thus, the curve of $y = \cos |x|$ is the curve of $y = \cos x$, too.

So putting in a graph, the curve of $y = \cos |x|$, we can put it the way below.

Fig. 0.2



So the curve of $y = \cos |x|$ is the same as the curve of $y = \cos x$.

And we can notice that the curve is symmetric about the y-axis. And in fact, if the curve of an equation is <u>symmetric about the *y*-axis</u>, the equation is said to be <u>even</u>.

If the curve is <u>not symmetric</u> (asymmetric) about the y-axis, the equation is said to be <u>odd</u>. And the same is true for a function, too.

So the cosine function $y = c(x) = \cos x$ is an ***even*** function, since its curve is symmetric about the y-axis. And we know the curve of the sine function $y = s(x) = \sin x$ is *asymmetric* about the y-axis. So the sine function $y = s(x) = \sin x$ is an ***odd*** function.

And we can check to see if a function is even or odd the way as follows.

If for instance, a function $y = f(x)$ is odd, we get $y = f(-x) = -f(x)$.

If however, $y = f(x)$ is even, we get $y = f(-x) = f(x)$.

For instance, the cosine function $y = C(x) = \cos x$ is an even function, because we have

$y = C(x) = \cos x = \cos(-x) = C(-x)$, that is, we have $y = C(x) = C(-x)$.

What then about $y = f(|x|)$?

It is even, because $y = f(|-x|) = f(|x|)$, because $|-x| = |x|$.

So for instance, if $y = f(x) = \sin x$, we get $y = f(|x|) = \sin |x|$, and thus, we get

$y = f(|-x|) = \sin |-x| = \sin |x|$.

And the same is true for an equation, too. So for instance,

$y = 3x^2$ is even, because we get $y = 3(-x)^2 = 3x^2$.

$y = -5|x^3|$ is even, because we get $y = -5|(-x)^3| = -5|-x^3| = -5|x^3|$.

$y = 2x$ is odd, because we get $y = 2(-x) = -2x$.

However, $y = 2|x^3| + x$ is neither even nor odd,
because we get $y = 2|(-x)^3| + (-x) = 2|x^3| - x \neq -(2|x^3| + x)$.

So we get neither of these: $y = f(-x) = -f(x)$ and $y = f(-x) = f(x)$.

And if a function or equation is even, the curve is symmetric about the y-axis.

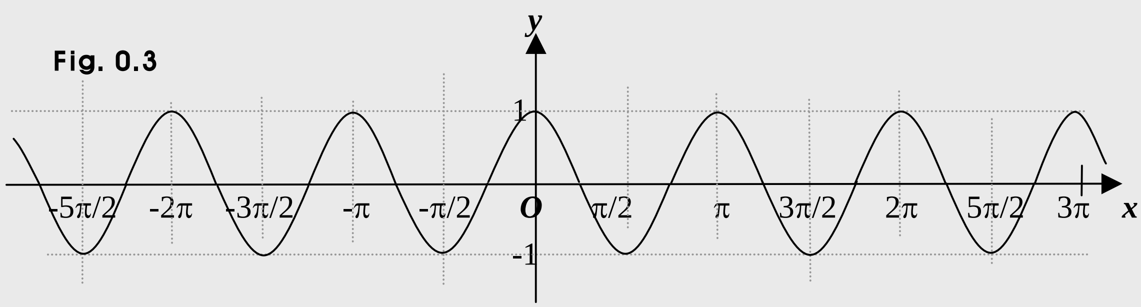
What then about $y = \cos 2x$?

The equation $\boldsymbol{y = \cos 2x}$ is even, because we get $\mathbf{\cos 2(-x) = \cos 2x}$. How?

Assuming $\boldsymbol{t = 2x}$, we can set $\mathbf{\cos 2x = \cos t}$.
Then, we get $\mathbf{2(-x) = -2x = -t}$, so we get $\mathbf{\cos (-2x) = \cos (-t) = \cos t = \cos 2x}$.

So given $\boldsymbol{y = \cos |2x|}$, we get $\boldsymbol{y = \cos 2x}$ for $\boldsymbol{x < 0}$, as well as for $\boldsymbol{x \geq 0}$.
Thus, the curve of $\boldsymbol{y = \cos |2x|}$ is the same as the curve of $\boldsymbol{y = \cos 2x}$.
And we know putting in a graph, the curve of the equation $\boldsymbol{y = \cos 2x}$, we get

Fig. 0.3



Note that the curve of an equation or function $\boldsymbol{y = h(|x|)}$ is symmetric about the $\boldsymbol{y}$-axis.

For instance, assuming: $\boldsymbol{h(|x|) = |x|^2 - 3|x| + 2}$, we get $\boldsymbol{y = |x|^2 - 3|x| + 2}$, and the curve of it is symmetric about the $\boldsymbol{y}$-axis.

And assuming: $\boldsymbol{h(|x|) = \sin |x| + 2|x| + 1}$, we get $\boldsymbol{y = \sin |x| + 2|x| + 1}$, and the curve of it is symmetric about the $\boldsymbol{y}$-axis. And of course, not all cosine functions are even functions.

For instance, assuming $\boldsymbol{y = g(x) = \cos (x - \pi/2)}$, we get

$\boldsymbol{g(-x) = \cos (-x - \pi/2) = \cos \{-(x + \pi/2)\} = \cos (x + \pi/2)}$.

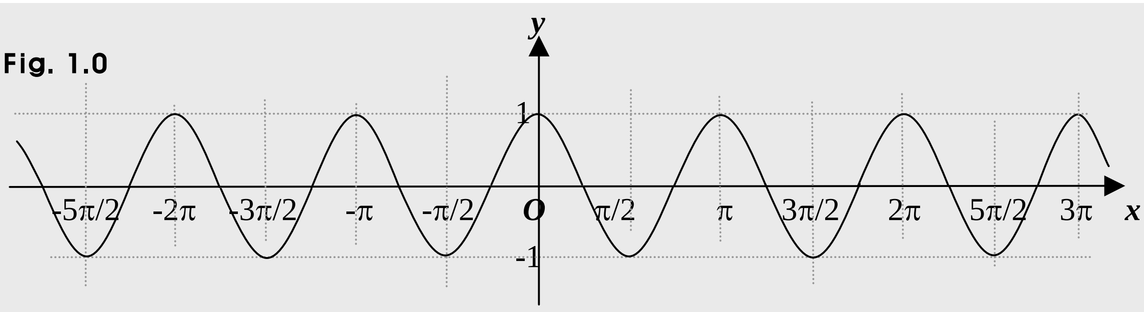
So we get $\boldsymbol{y = g(x) \neq g(-x)}$.

Suggestions or Solutions
To the Problem in the Example 1

Put in a graph the equation as follows: $|y| = \cos 2x$.

To begin with, putting in the ***x-y*** plane, the curve of the equation $y = \cos 2x$, we can put it the way below.

Fig. 1.0



What then about the curve of $|y| = \cos 2x$?

In that case, we want to consider two cases, one is $y \geq 0$, and the other is $y < 0$.   Why?

Though looking awkward, $|y| = \cos 2x$ can be put this way, too: $t = \cos 2x$ where $t = |y|$.

And we know $|y| \geq 0$ for all y.

So even if $y < 0$, that is, y gets a negative value, we still get $t > 0$, that is, t gets a positive value.

So for instance, even if $y = -1/2$, we need to get $t = 1/2$, because $|-1/2| = 1/2$.

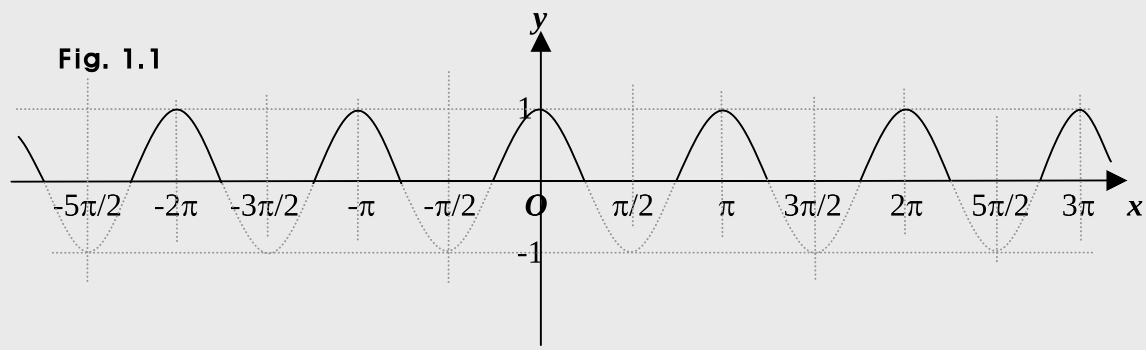
So given $|y| = \cos 2x$ for $y \geq 0$, we can just set $y = \cos 2x$.

If however, $y < 0$, we want to set $-y = \cos 2x$ because $-y > 0$, since $|y| \geq 0$ in $|y| = \cos 2x$.

Thus, we get $y = \cos 2x$ for $y \geq 0$, and we get $y = -\cos 2x$ for $y < 0$.

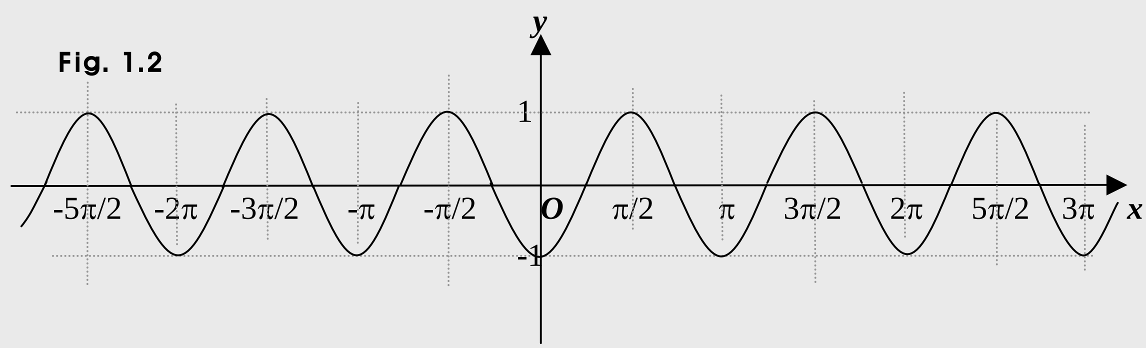
And taking care of first, the curve of $y = \cos 2x$ for $y \geq 0$, we can put it this way:

Fig. 1.1



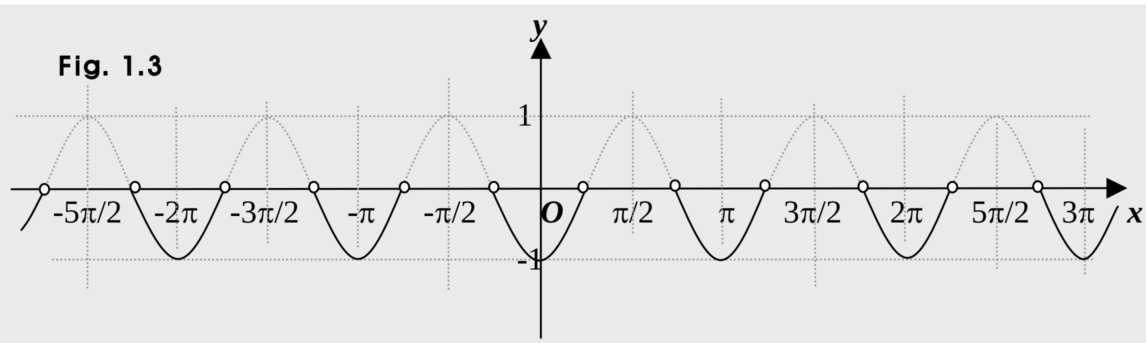
Next, we can put in a graph, the curve of $y = -\cos 2x$ the way below.

Fig. 1.2



So next, moving on to the curve of $y = -\cos 2x$ for $y < 0$, we can put it this way:

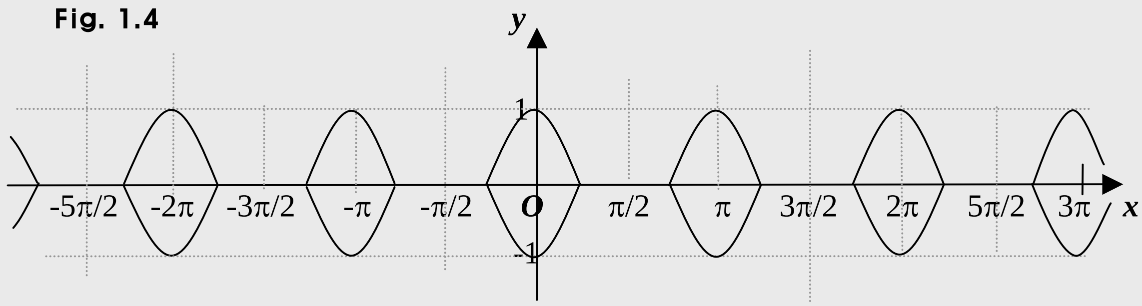
Fig. 1.3



Note that all the points $\left(\frac{(2\boldsymbol{n}+1)\pi}{4}, \mathbf{0}\right)$ for n integer, do not belong to the curve above.

Thus, putting in a graph, the curve of $|y| = \cos 2x$, we can put it the way as follows.

Fig. 1.4



Notice that the curve is symmetric about the ***x***-axis.

In fact, the curve of an equation or function $|y| = h(x)$ is symmetric about the ***x***-axis.

For instance, assuming $h(x) = x^2 - 3x + 2$, we can get $|y| = x^2 - 3x + 2$, and the curve of it is symmetric about the ***x***-axis.

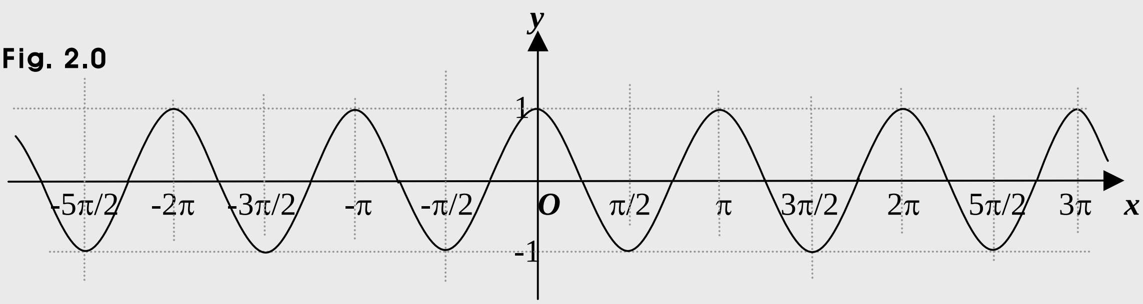
And assuming $h(x) = \sin x + 2x + 1$, we can get $|y| = \sin x + 2x + 1$, and the curve of it is symmetric about the ***x***-axis.

Suggestions or Solutions
To the Problem in the Example 2

Put in a graph the equation as follows: $|y| = \cos |2x|$.

To begin with, putting in the x-y plane, the curve of the equation $y = \cos 2x$, we can put it the way below.

Fig. 2.0



Next, we know $|y| \geq 0$ for all y, and $|2x| \geq 0$ for all x.

So we want to consider four different cases as follows.

First, assuming $x \geq 0$, and $y \geq 0$, we get $y = \cos 2x$.

Next, assuming $x \geq 0$, and $y < 0$, we get $-y = \cos 2x$, so we get $y = -\cos 2x$.

Next, assuming $x < 0$, and $y < 0$, we get $-y = \cos (-2x)$, so we get $y = -\cos 2x$.

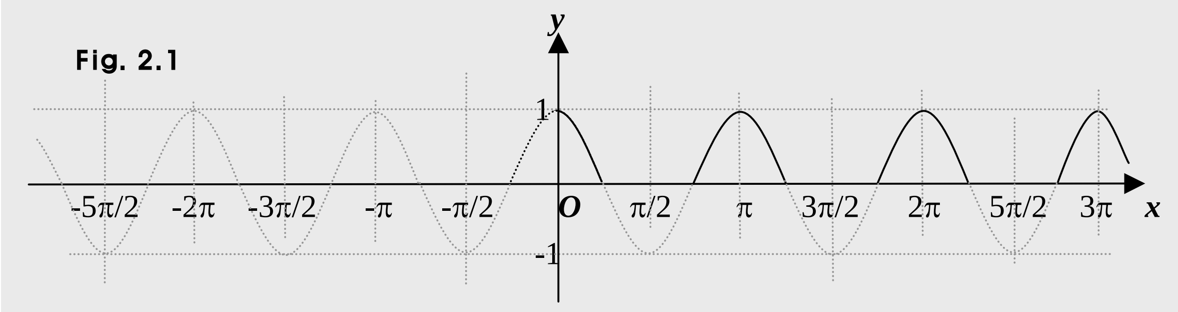
And next, assuming $x < 0$, and $y \geq 0$, we get $y = \cos (-2x)$, so we get $y = \cos 2x$.

Thus in sum, we have these:

$y = \cos 2x$ for $x \geq 0$, and $y \geq 0$.	$y = -\cos 2x$ for $x < 0$, and $y < 0$.
$y = -\cos 2x$ for $x \geq 0$, and $y < 0$.	$y = \cos 2x$ for $x < 0$, and $y \geq 0$.

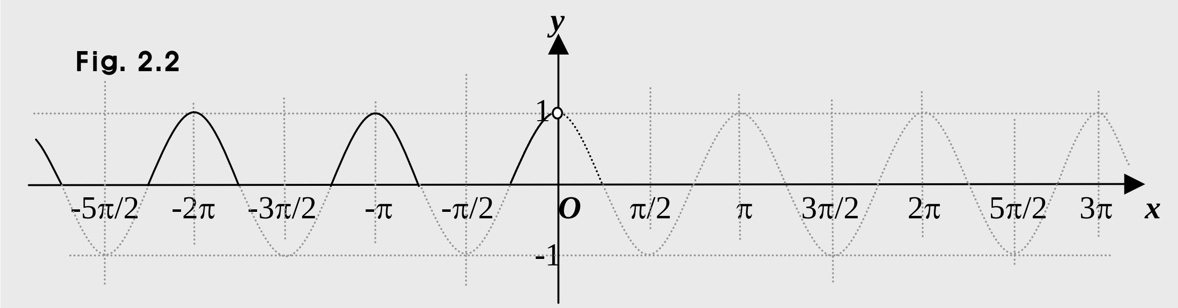
So beginning with the curve of $y = \cos 2x$ for $x \geq 0$, and $y \geq 0$, we get

Fig. 2.1



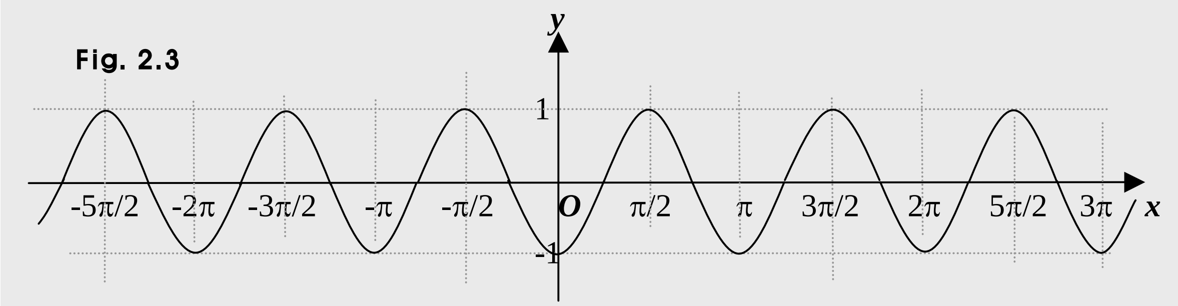
Next, moving on to the curve of $y = \cos 2x$ for $x < 0$, and $y \geq 0$, we get

Fig. 2.2



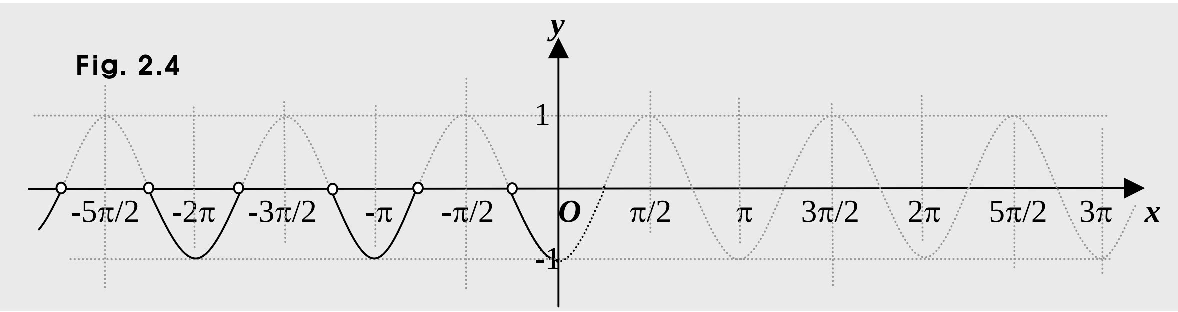
Next, we can put in a graph, the curve of $y = -\cos 2x$ the way below.

Fig. 2.3



So moving on to the curve of $y = -\cos 2x$ for $x < 0$, and $y < 0$, we get

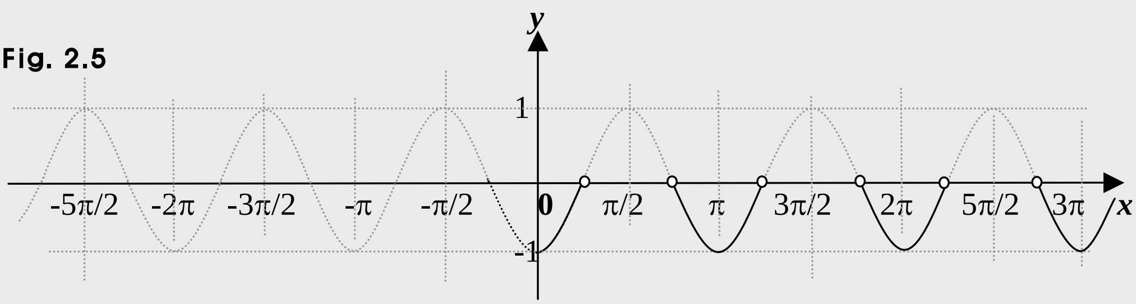
Fig. 2.4



Note that all the points $(-(2n + 1)\pi/4, 0)$ for n integer ≥ 0, do not belong to the curve.

Next, moving on to the curve of $\boldsymbol{y = -\cos 2x}$ for $\boldsymbol{x \geq 0}$, and $\boldsymbol{y < 0}$, we get this:

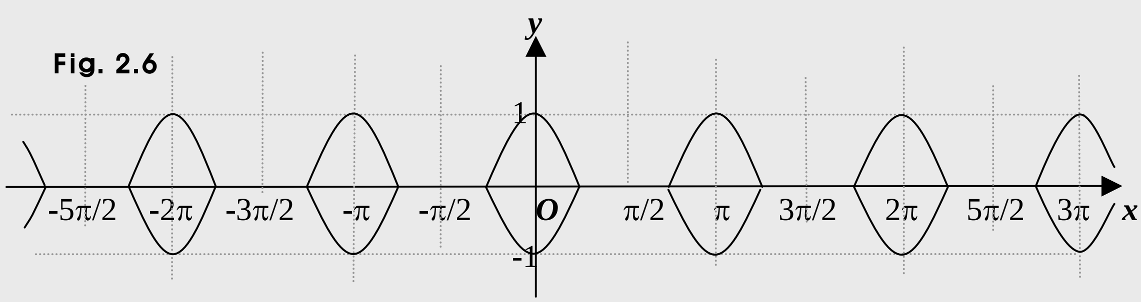
Fig. 2.5



(Note that all the points at $\boldsymbol{(\frac{(2n+1)\pi}{4}, 0)}$ for $\boldsymbol{n}$ integer $\geq \boldsymbol{0}$, do not belong to the curve above.)

Thus, putting in a graph, the curve of $\boldsymbol{|y| = \cos |2x|}$, we can put it the way as follows.

Fig. 2.6



Notice that the curve is symmetric about the origin, as well as the $\boldsymbol{x}$-axis and the $\boldsymbol{y}$-axis.

In fact, the curve of an equation or function $\boldsymbol{|y| = h(|x|)}$ is

symmetric about the origin, as well as the $\boldsymbol{x}$-axis and the $\boldsymbol{y}$-axis.

For instance, assuming $\boldsymbol{h(|x|) = |x|^2 - 3|x| + 2}$, we get $\boldsymbol{|y| = |x|^2 - 3|x| + 2}$, and the curve of it is symmetric about the origin, as well as the $\boldsymbol{x}$-axis and the $\boldsymbol{y}$-axis.

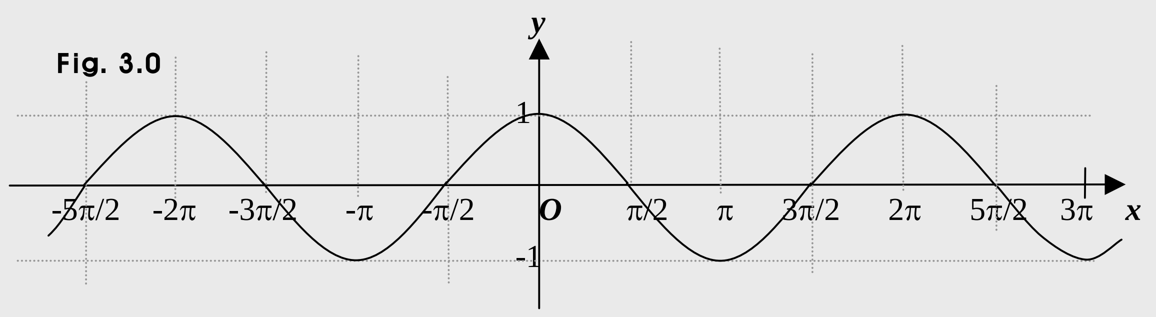
And assuming $\boldsymbol{h(|x|) = \sin |x| + 2|x| + 1}$, we get $\boldsymbol{|y| = \sin |x| + 2|x| + 1}$, and the curve of it is symmetric about the origin, as well as the $\boldsymbol{x}$-axis and the $\boldsymbol{y}$-axis.

Suggestions or Solutions
To the Problem in the Example 3

Put in a graph the equation as follows: $y = \cos x + \cos |x|$.

To begin with, we can put in the ***x-y*** plane, the curve of $y = \cos x$, the way below.

Fig. 3.0

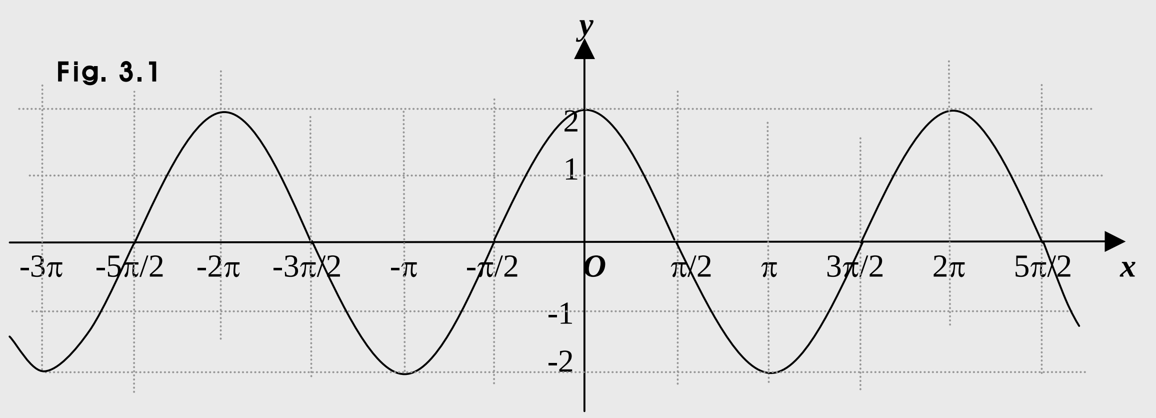


And in fact, putting in a graph, the curve of $y = \cos |x|$, we get the same one as above.

That's because $\cos |x| = \cos x$.

So the curve of $y = \cos x + \cos |x|$ is the same as the curve of $y = 2\cos x$.

Fig. 3.1



Thus, the curve above is the curve of the equation $y = \cos x + \cos |x|$.

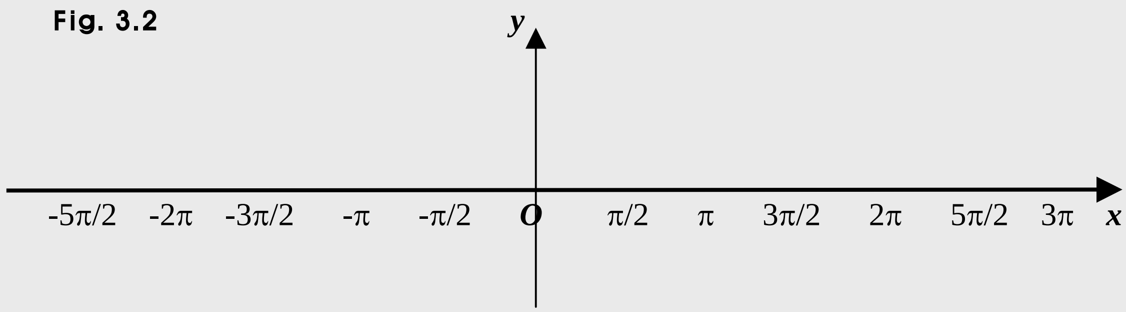
What then about the curve of $y = \cos x - \cos |x|$?

We know **cos** $\boldsymbol{x}$ **= cos** $|\boldsymbol{x}|$.

So we get **cos** $\boldsymbol{x}$ **– cos** $|\boldsymbol{x}|$ **= 0**. That is, we get $\boldsymbol{y}$ **= 0**.

Thus, the curve of $\boldsymbol{y}$ **= cos** $\boldsymbol{x}$ **– cos** $|\boldsymbol{x}|$ is the $\boldsymbol{x}$-axis, itself, which is thus, as follows.

Fig. 3.2

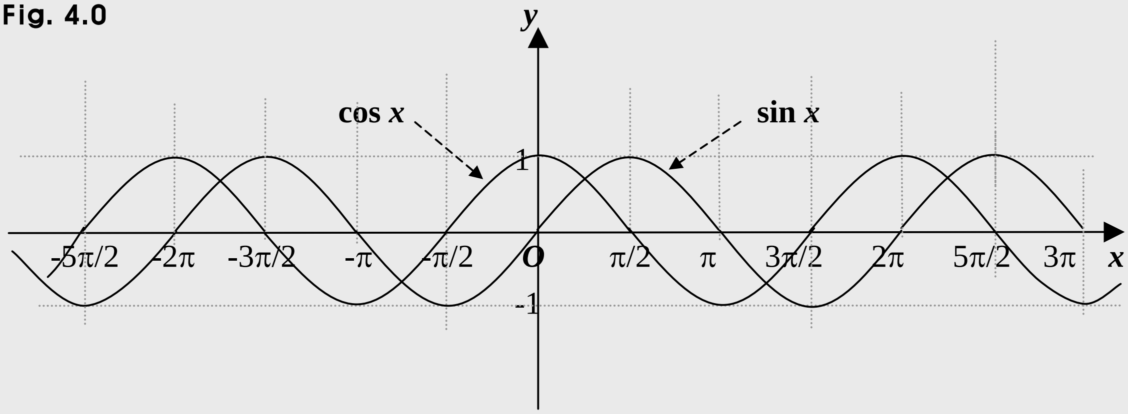


Suggestions or Solutions
To the Problem in the Example 4

Put in a graph the equation as follows: $y = \cos x + \sin x$.

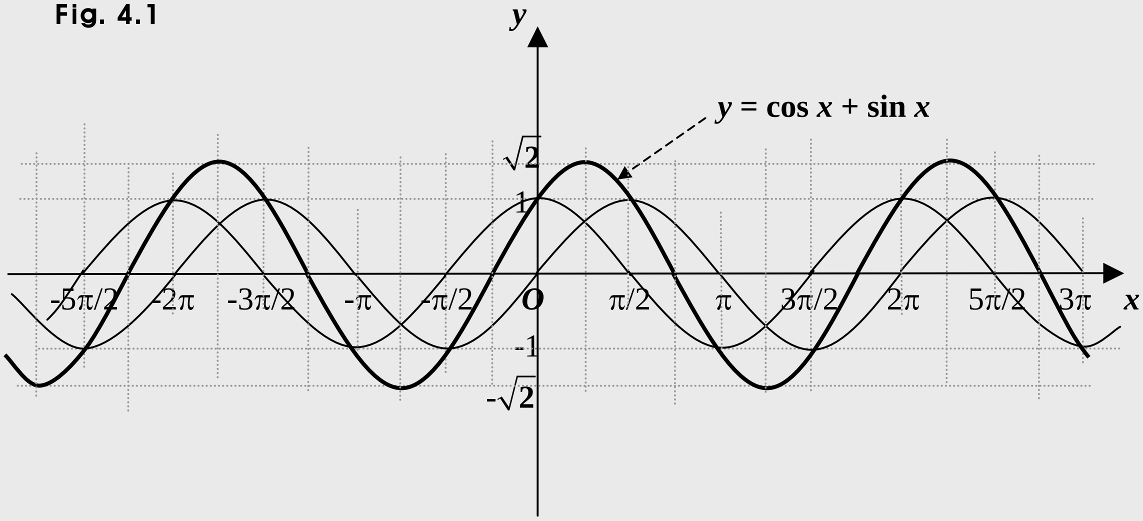
We can put in one ***x-y*** plane, the curves of $\boldsymbol{y = \cos x}$ and $\boldsymbol{y = \sin x}$, the way below.

Fig. 4.0



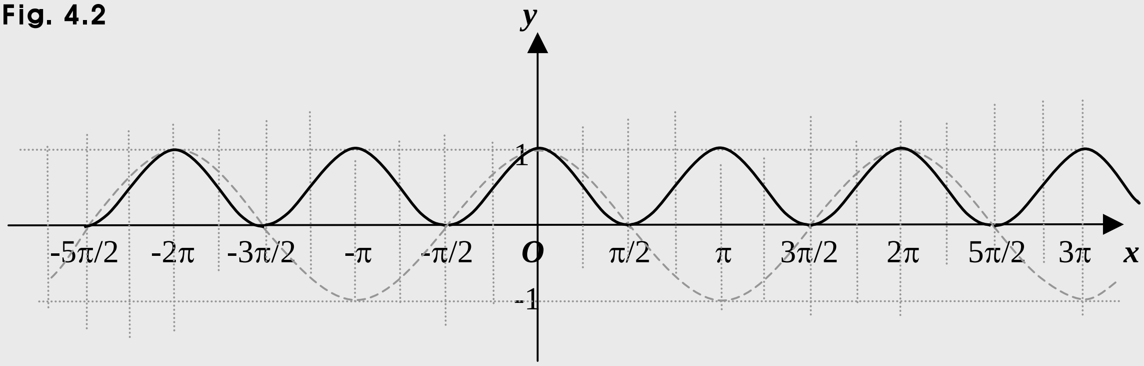
And next, adding together the two curves above, we get another curve. And we can put in one graph, all the three curves the way below.

Fig. 4.1



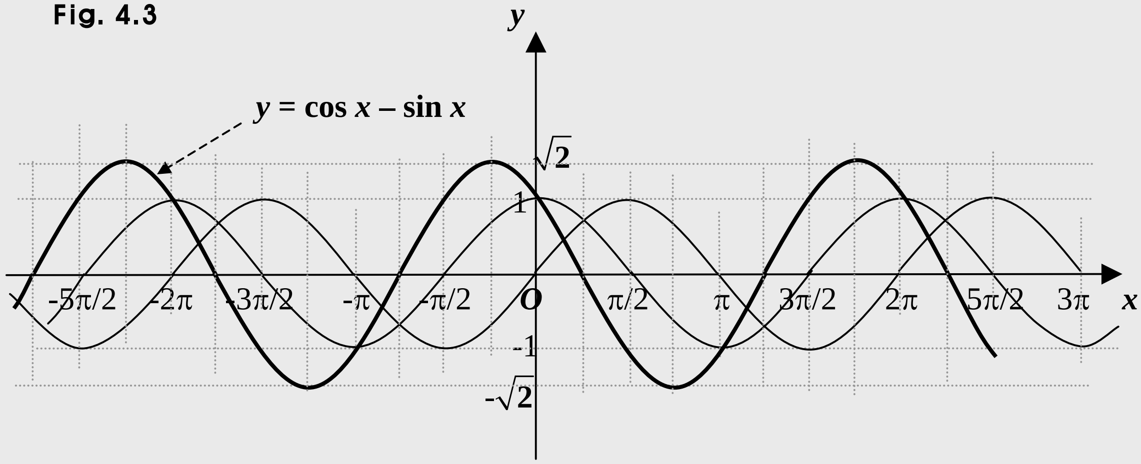
And we can expect the curve of $\boldsymbol{y = \cos^2 x}$ to be made the way below.

Fig. 4.2



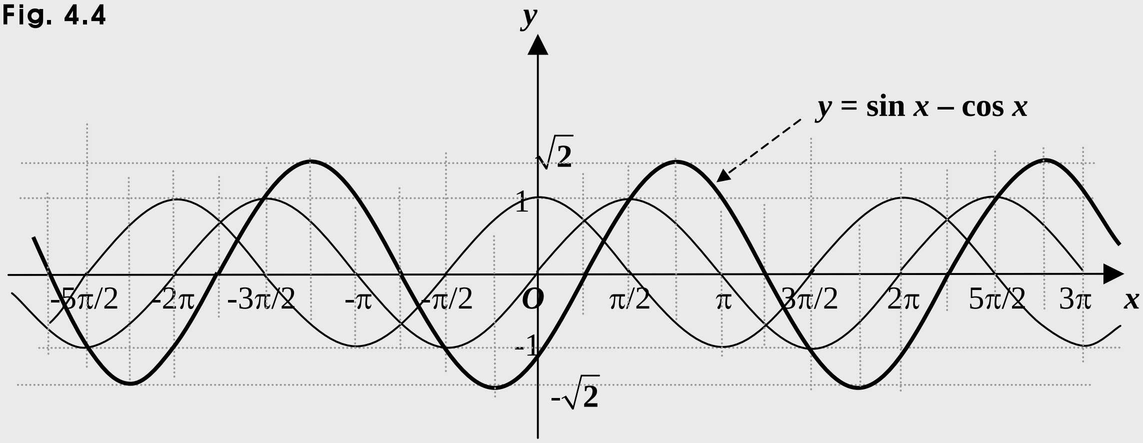
And we can put in a graph, the the curve of ***y* = cos *x* – sin *x*** the way as follows.

Fig. 4.3



Also, we can put in a graph, the the curve of ***y* = sin *x* – cos *x*** the way below.

Fig. 4.4



And assuming $\boldsymbol{f(x) = \sin x + \cos x}$, $\boldsymbol{g(x) = \cos x - \sin x}$, and $\boldsymbol{h(x) = \sin x - \cos x}$, we can notice that $\boldsymbol{f}$, $\boldsymbol{g}$, and $\boldsymbol{h}$ have the same amplitude and period.

The amplitude is $\sqrt{\mathbf{2}}$, and the period is 2π.

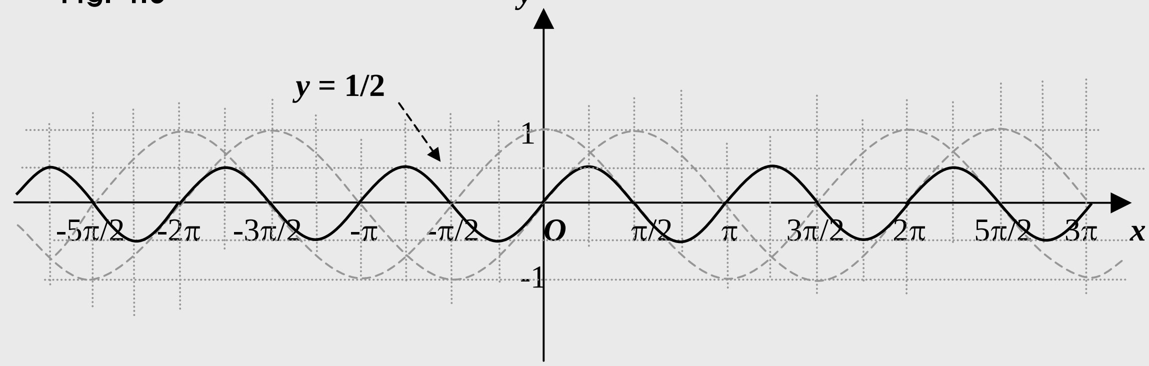
And in fact, the curves themselves of $\boldsymbol{f}$, $\boldsymbol{g}$, and $\boldsymbol{h}$ are actually the same.

The only difference between the three is in the phase.

That is to say that we have $\boldsymbol{f(x + \pi) = g(x)}$, $\boldsymbol{f(x) = g(x - \pi/2)}$, and $\boldsymbol{g(x - \pi) = h(x)}$.

Besides, we can put in a graph, the the curve of $\boldsymbol{y = \cos x \sin x}$ the way below.

Fig. 4.5



And in fact, we have $\mathbf{\sin 2}\boldsymbol{x}\mathbf{ = 2 \cos }\boldsymbol{x}\mathbf{ \sin }\boldsymbol{x}$. That is, $\boldsymbol{\cos x \sin x = \frac{1}{2} \sin 2x}$.