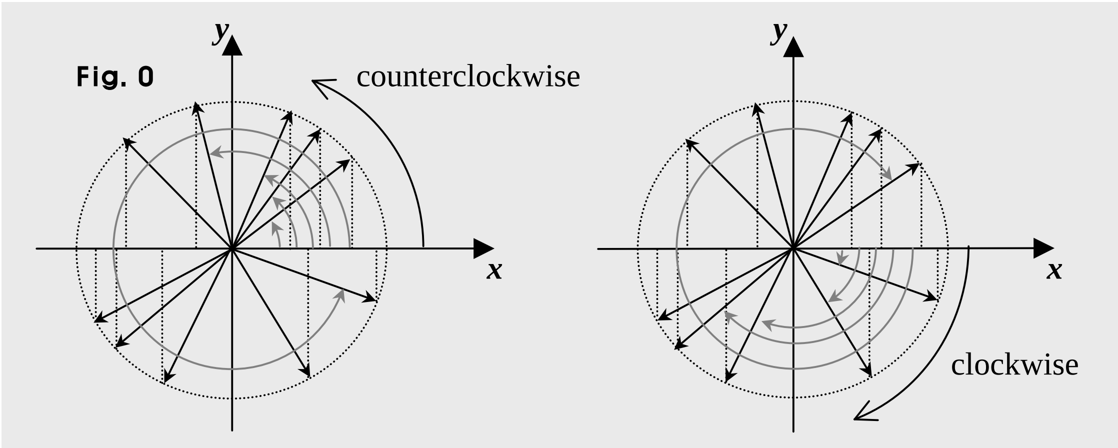
Examples in Tangent Functions

Click this to start.

Examples in Tangent Functions

Fig. 0



In coordinate trigonometry, the vector turning makes the angles. If turning clockwise, it makes angles negative, and if counterclockwise, it makes angles positive.

And of course, if no turning, the angle made is 0. So the angle can be 0^o or any angle positive or negative. In each right triangle, the vector is of length 1, and is the hypotenuse. And $\boldsymbol{(x, y)}$ is the terminal point, so $\boldsymbol{x}$ is the adjacent, and $\boldsymbol{y}$ is the opposite.

Thus, assuming θ is a governing angle, we get $\mathbf{tan}\ \theta = \dfrac{y}{x}$.

So if the vector is in the first quadrant, $\mathbf{tan}\ \theta > \mathbf{0}$, since both $\boldsymbol{x}$ and $\boldsymbol{y}$ are $> \mathbf{0}$.
In the second, $\mathbf{tan}\ \theta < \mathbf{0}$, since $\boldsymbol{x} < \mathbf{0}$ but $\boldsymbol{y} > \mathbf{0}$.
In the third, $\mathbf{tan}\ \theta > \mathbf{0}$, too, because both $\boldsymbol{x}$ and $\boldsymbol{y}$ are $< \mathbf{0}$.
And in the fourth, $\mathbf{tan}\ \theta < \mathbf{0}$, since $\boldsymbol{x} > \mathbf{0}$ but $\boldsymbol{y} < \mathbf{0}$.

Put in a graph the curve of each of the equations below.

0. $y = \frac{1}{2}\tan|\frac{x}{2}|$ 1. $|y| = \frac{1}{2}\tan\frac{x}{2}$ and $y = |\frac{1}{2}\tan\frac{x}{2}|$

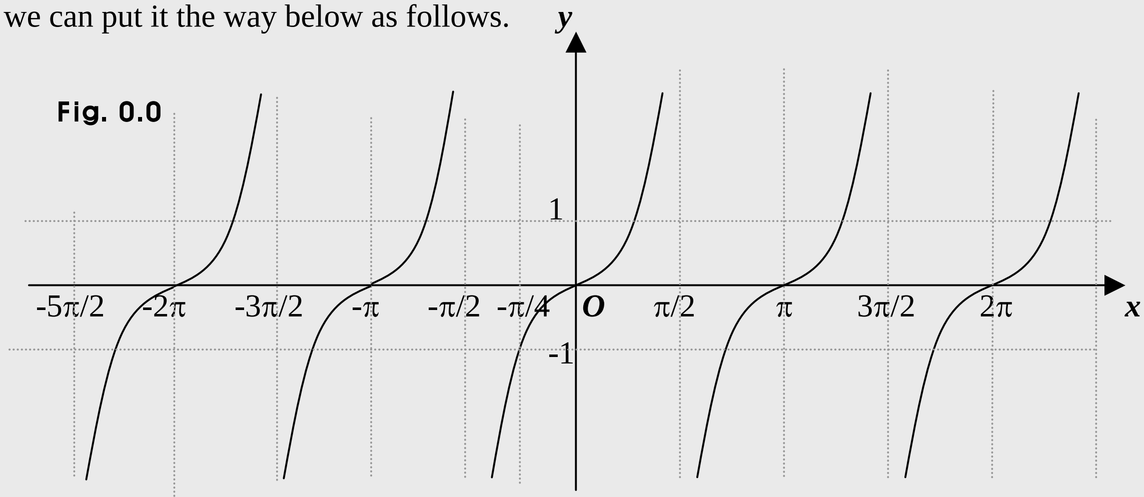
2. $|y| = \frac{1}{2}\tan|\frac{x}{2}|$ 3. $y = \frac{1}{2}(\tan\frac{x}{2} + \tan|\frac{x}{2}|)$ 4. $|y| = \frac{1}{2}(\tan\frac{x}{2} + \tan|\frac{x}{2}|)$

Suggestions or Solutions
To the Problem in the Example 0

Put in a graph the equation as follows: $y = \frac{1}{2}\tan\left|\frac{x}{2}\right|$.

To begin with, putting in the ***x-y*** plane, the curve of the tangent function $y = f(x) = \tan x$, we can put it the way below as follows.

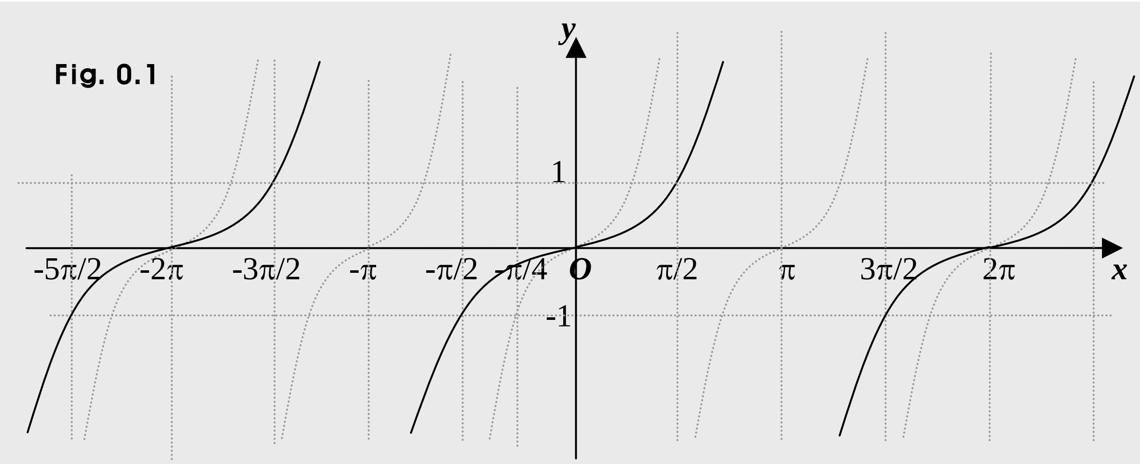
Fig. 0.0



The curve above is in fact, the same as the curve of an equation $y = \tan x$.
Notice that when $x = n\pi + \pi/4 = (4n + 1)\pi/4$, we get $y = 1$. That is, $\tan (4n + 1)\pi/4 = 1$.
And the curve of an equation $y = \tan\frac{x}{2}$ is the curve of a function $y = g(x) = \tan\frac{x}{2}$, too.

So putting in a graph, the curve of the equation $y = \tan\frac{x}{2}$, we can put it this way:

Fig. 0.1



And we can see that the period is 2π, and that the frequency is 1/2.

The frequency says how many times the periodic part appears in the basic period. That is to say that the frequency is the basic period over the particular period.

The basic period of the tangent, that is, the period of **tan *x*** is π.

And the particular period, that is, the period of $\mathbf{tan}\,\frac{\boldsymbol{x}}{\mathbf{2}}$ is 2π.

So the frequency of the particular tangent, that is, the frequency of $\mathbf{tan}\,\frac{\boldsymbol{x}}{\mathbf{2}}$ is $\frac{\pi}{2\pi} = \frac{1}{2}$.

And normally, setting ***y* = tan *wx***, we mean ***w*** is the frequency.

So looking at $\mathbf{tan}\,\frac{\boldsymbol{x}}{\mathbf{2}}$, we can readily see that the frequency is $\frac{1}{2}$.

What then about the curve of $\boldsymbol{y} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{tan}\,\frac{\boldsymbol{x}}{\mathbf{2}}$?

For every value of ***x***, the value of $\frac{\mathbf{1}}{\mathbf{2}}\mathbf{tan}\,\frac{\boldsymbol{x}}{\mathbf{2}}$ is half the value of $\mathbf{tan}\,\frac{\boldsymbol{x}}{\mathbf{2}}$.

So we can put in a graph, the curve of $\boldsymbol{y} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{tan}\,\frac{\boldsymbol{x}}{\mathbf{2}}$ the way below.

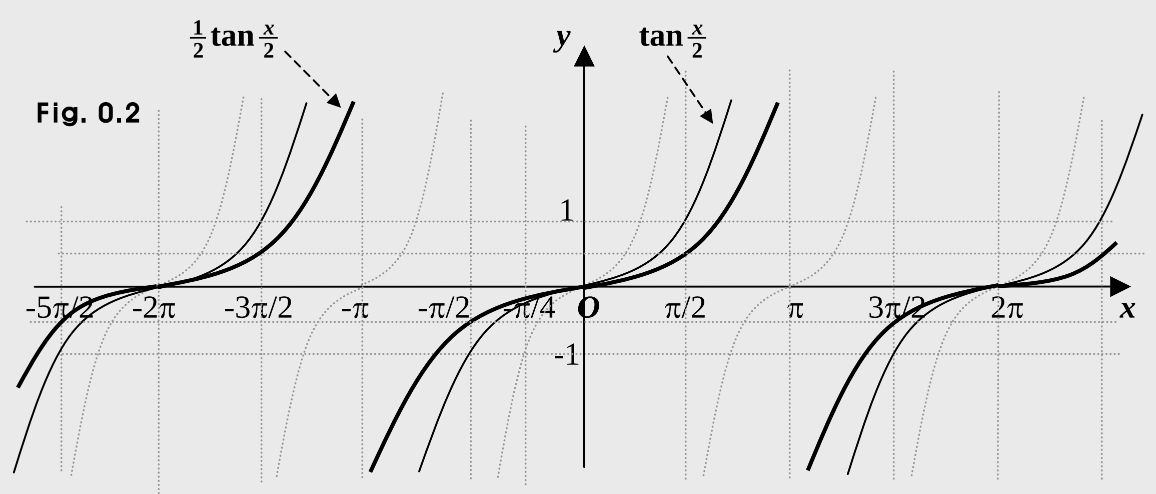


Fig. 0.2

What then, about $\boldsymbol{y} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{tan}\left|\frac{\boldsymbol{x}}{\mathbf{2}}\right|$?

In that case, we want to consider two cases, one is $\boldsymbol{x} \geq \mathbf{0}$, and the other is $\boldsymbol{x} < \mathbf{0}$.   Why?

We can put the equation this way, too: $\boldsymbol{y} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{tan}\frac{\mathbf{1}}{\mathbf{2}}|\,\boldsymbol{x}\,|$.

And since $\frac{\mathbf{1}}{\mathbf{2}}$ is just a number, we may want to just consider this equation: $\boldsymbol{y} = \mathbf{tan}\ |\boldsymbol{x}|$.

Also, although looking awkward, $\mathbf{tan}\ |\boldsymbol{x}|$ can be put this way, too: $\mathbf{tan}\ \boldsymbol{t}$ where $\boldsymbol{t} = |\boldsymbol{x}|$.
And we know $|\boldsymbol{x}| \geq \mathbf{0}$ for all $\boldsymbol{x}$.
So even if $\boldsymbol{x} < \mathbf{0}$, that is, $\boldsymbol{x}$ gets a negative value, we still get $\boldsymbol{t} > \mathbf{0}$, that is, $\boldsymbol{t}$ gets a positive value.

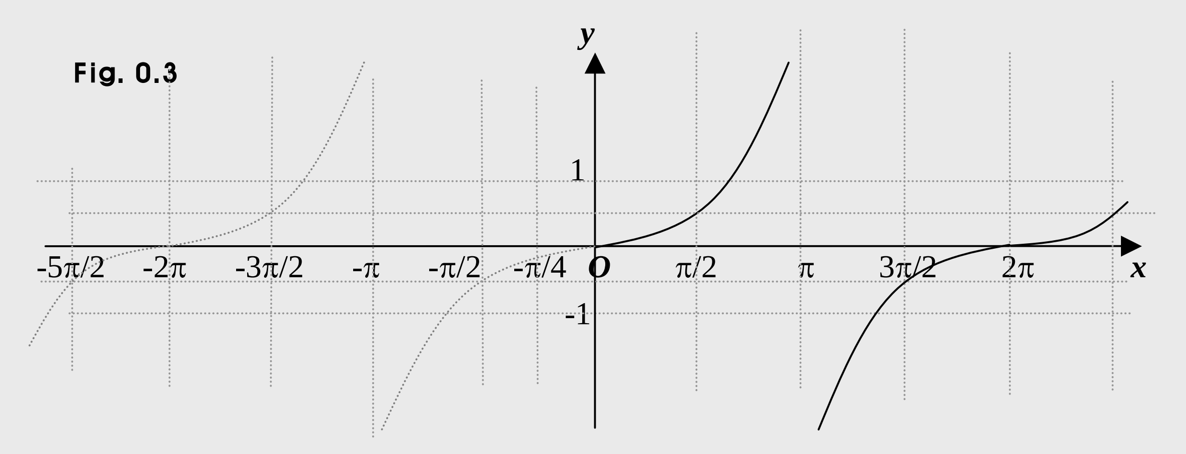
Thus, for instance, even if $\boldsymbol{x} = \mathbf{-\pi/4}$, we need to get $\mathbf{tan}\ \boldsymbol{\pi}\mathbf{/4}$, since $|\mathbf{-\pi/4}| = \boldsymbol{\pi}\mathbf{/4}$.
So given $\boldsymbol{y} = \mathbf{tan}\ |\boldsymbol{x}|$ for $\boldsymbol{x} \geq \mathbf{0}$, we can just set $\boldsymbol{y} = \mathbf{tan}\ \boldsymbol{x}$.

If however, $\boldsymbol{x} < \mathbf{0}$, we want to set $\boldsymbol{y} = \mathbf{tan}\ (\mathbf{-}\boldsymbol{x})$ because $\mathbf{-}\boldsymbol{x} > \mathbf{0}$, since $|\boldsymbol{x}| \geq \mathbf{0}$ in $\mathbf{tan}\ |\boldsymbol{x}|$.
And we have a trig identity $\mathbf{tan}\ (\mathbf{-}\boldsymbol{x}) = \mathbf{-tan}\ \boldsymbol{x}$.

Thus, given $\boldsymbol{y} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{tan}\frac{\mathbf{1}}{\mathbf{2}}|\,\boldsymbol{x}\,|$, we get $\boldsymbol{y} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{tan}\frac{\boldsymbol{x}}{\mathbf{2}}$ for $\boldsymbol{x} \geq \mathbf{0}$, and $\boldsymbol{y} = -\frac{\mathbf{1}}{\mathbf{2}}\mathbf{tan}\frac{\boldsymbol{x}}{\mathbf{2}}$ for $\boldsymbol{x} < \mathbf{0}$.

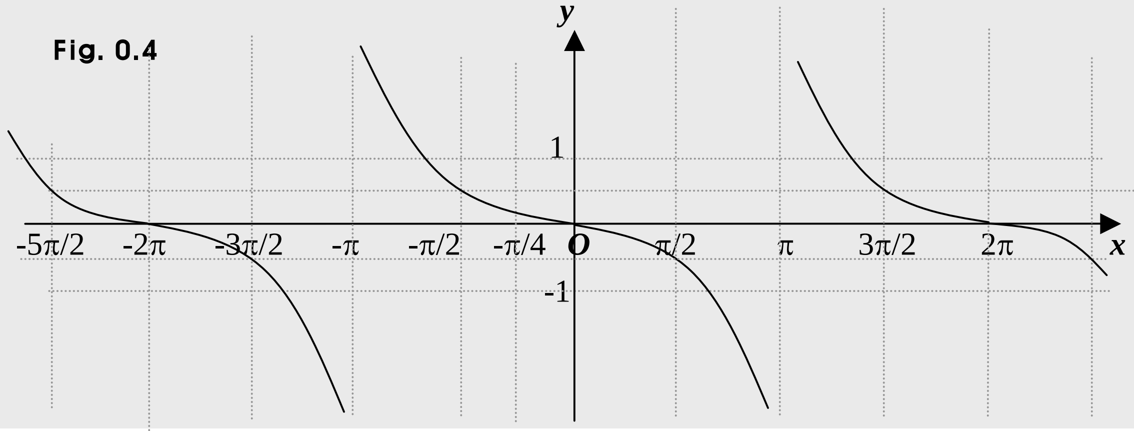
So taking care of first, the curve of $\boldsymbol{y} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{tan}\frac{\boldsymbol{x}}{\mathbf{2}}$ for $\boldsymbol{x} \geq \mathbf{0}$, we can put it the way below.

Fig. 0.3



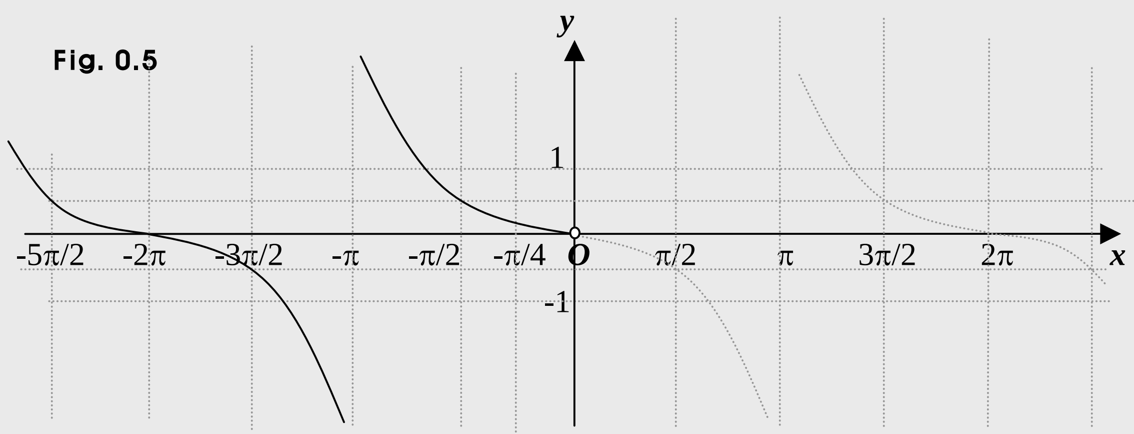
Next, putting in a graph, the curve of $\boldsymbol{y} = -\frac{\mathbf{1}}{\mathbf{2}}\mathbf{tan}\frac{\mathbf{1}}{\mathbf{2}}|\,\boldsymbol{x}\,|$, we can put it the way below.

Fig. 0.4



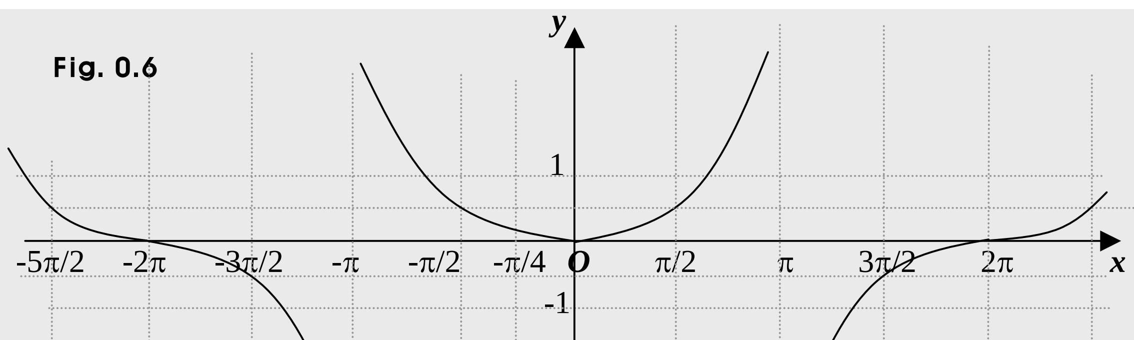
So moving on to the curve of $y = -\dfrac{1}{2}\tan\dfrac{x}{2}$ for $x < 0$, we can put it the way below.

Fig. 0.5



Thus, putting in a graph, the curve of $y = \dfrac{1}{2}\tan\left|\dfrac{x}{2}\right|$ we can put it the way as follows.

Fig. 0.6



Notice that the curve is symmetric about the y-axis.

In fact, the curve of an equation or a function $y = f(|x|)$ is symmetric about the y-axis.

Suggestions or Solutions
To the Problem in the Example 1

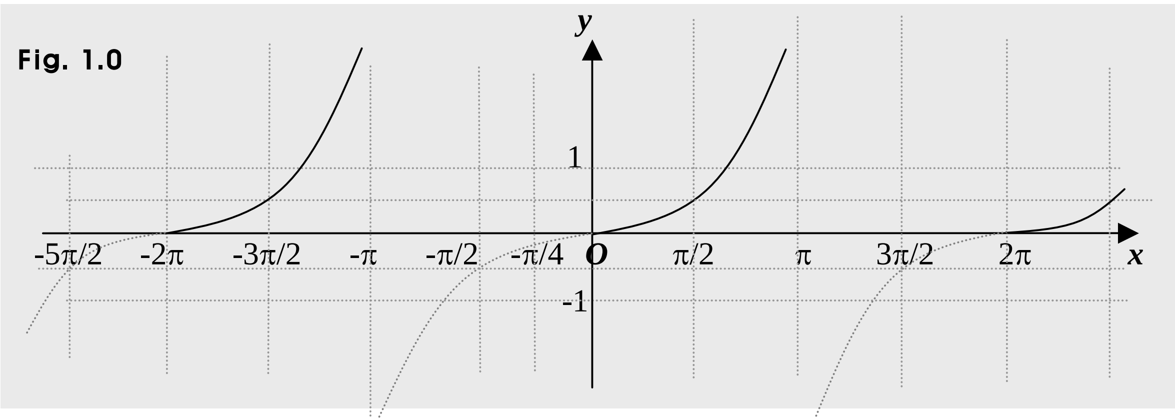
Put in a graph the equations as follows: $|y| = \mathbf{\frac{1}{2}\tan\frac{x}{2}}$ and $y = |\mathbf{\frac{1}{2}\tan\frac{x}{2}}|$.

So what's the difference between the two equations?

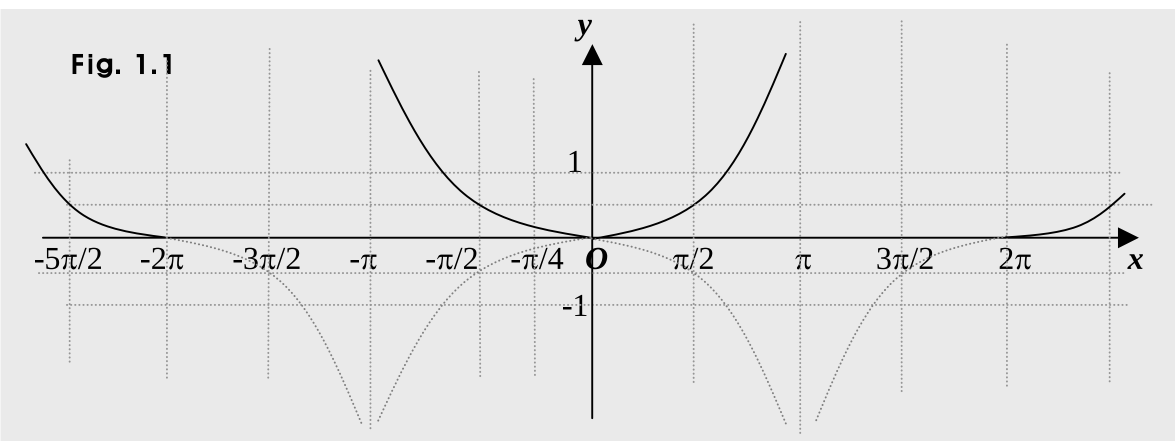
Other than the looks, they are the same. So both are the same.
Thus for instance, an equation $y = |\tan x|$ is the same as an equation $|y| = \tan x$.

And we know $|y| \geq 0$. So we get $y = \frac{1}{2}\tan\frac{x}{2}$ for $y \geq 0$, and $y = -\frac{1}{2}\tan\frac{x}{2}$ for $y < 0$.

And putting in a graph, the curve of $y = \frac{1}{2}\tan\frac{x}{2}$ for $y \geq 0$, we can put it the way below.



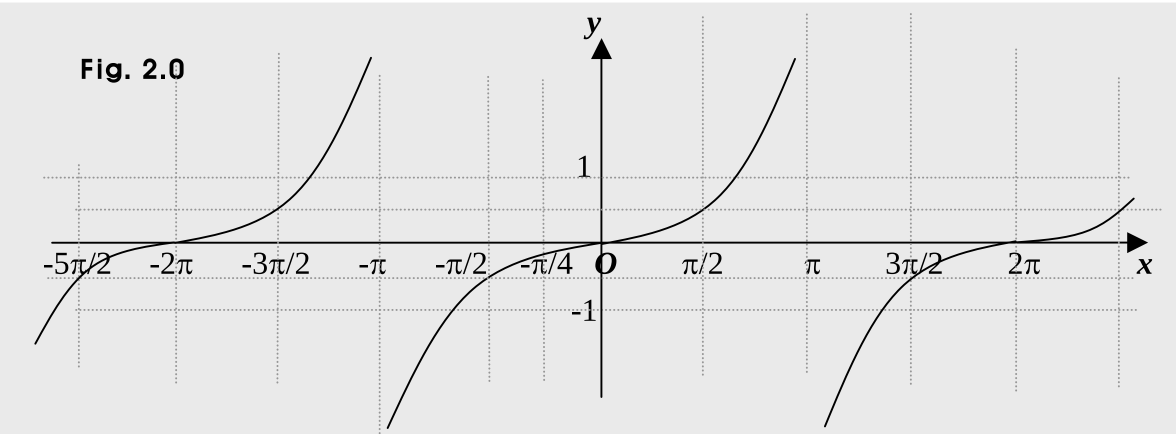
And we know the curve is symmetric about the y-axis.
So we can put the curve of $|y| = \frac{1}{2}\tan\frac{x}{2}$ or $y = |\frac{1}{2}\tan\frac{x}{2}|$, the way as follows.



Suggestions or Solutions
To the Problem in the Example 2

Put in a graph the equations as follows: $|y| = \frac{1}{2}\tan\left|\frac{x}{2}\right|$.

To begin with, putting in the ***x-y*** plane, the curve of the equation $y = \frac{1}{2}\tan\frac{x}{2}$, we can put it the way below.



Next, we know $|y| \geq 0$ for all y, and $|2x| \geq 0$ for all x.
So we want to consider four different cases as follows.

First, assuming $x \geq 0$, and $y \geq 0$, we get $y = \frac{1}{2}\tan\frac{x}{2}$.

Next, assuming $x \geq 0$, and $y < 0$, we get $-y = \frac{1}{2}\tan\frac{x}{2}$, so we get $y = -\frac{1}{2}\tan\frac{x}{2}$.

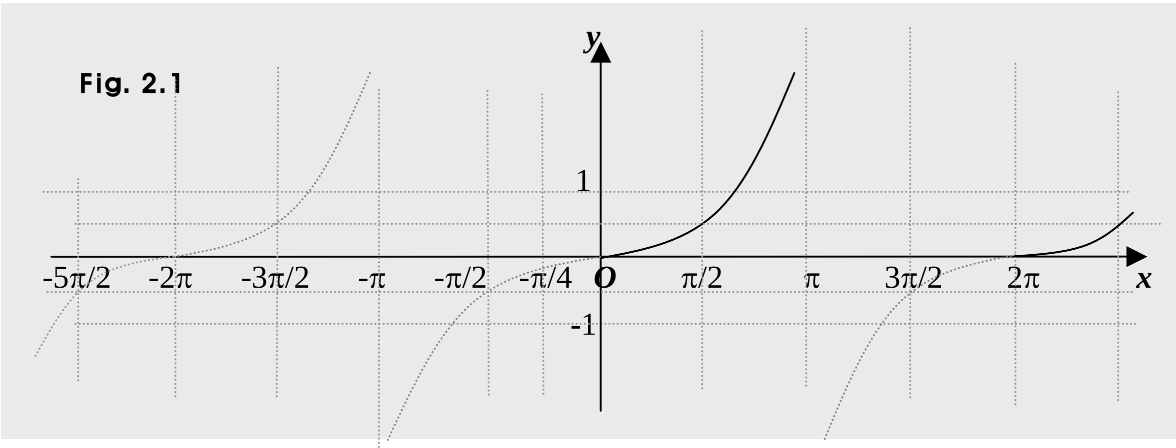
Next, assuming $x < 0$, and $y < 0$, we get $-y = \frac{1}{2}\tan(-\frac{x}{2}) = -\frac{1}{2}\tan\frac{x}{2}$, so we get $y = \frac{1}{2}\tan\frac{x}{2}$.

And next, assuming $x < 0$, and $y \geq 0$, we get $y = \frac{1}{2}\tan(-\frac{x}{2})$, so we get $y = -\frac{1}{2}\tan\frac{x}{2}$.

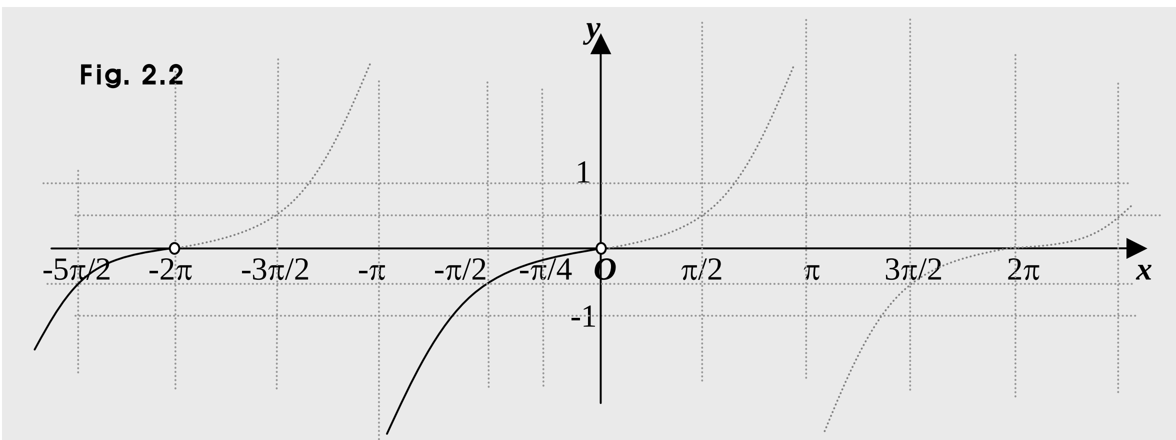
Thus in sum, we have these:

$y = \frac{1}{2}\tan\frac{x}{2}$ for $x \geq 0$, and $y \geq 0$.	$y = \frac{1}{2}\tan\frac{x}{2}$ for $x < 0$, and $y < 0$.
$y = -\frac{1}{2}\tan\frac{x}{2}$ for $x \geq 0$, and $y < 0$.	$y = -\frac{1}{2}\tan\frac{x}{2}$ for $x < 0$, and $y \geq 0$.

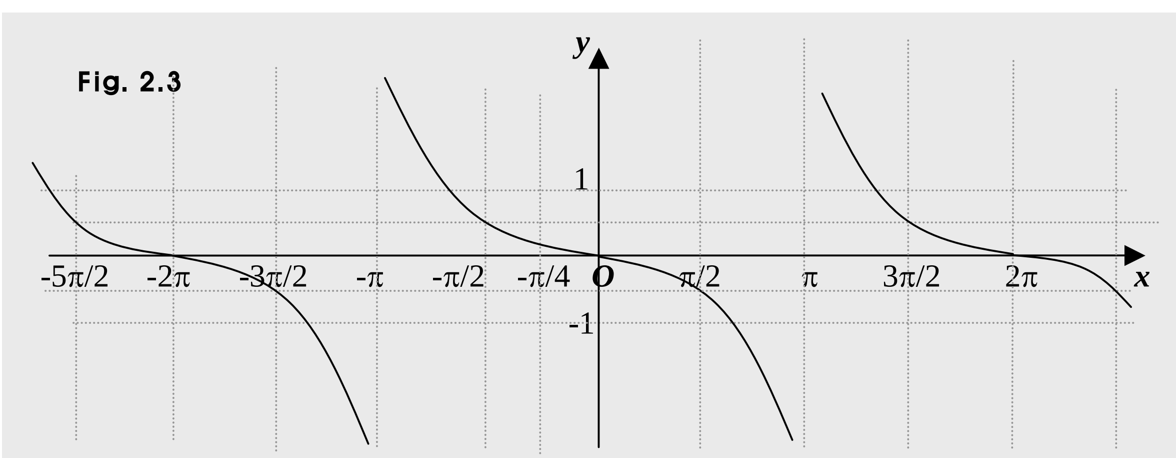
So beginning with the curve of $y = \frac{1}{2}\tan\frac{x}{2}$ for $x \geq 0$, and $y \geq 0$, we can get this:



Next, moving on to the curve of $y = \frac{1}{2}\tan\frac{x}{2}$ for $x < 0$, and $y \geq 0$, we can get this:

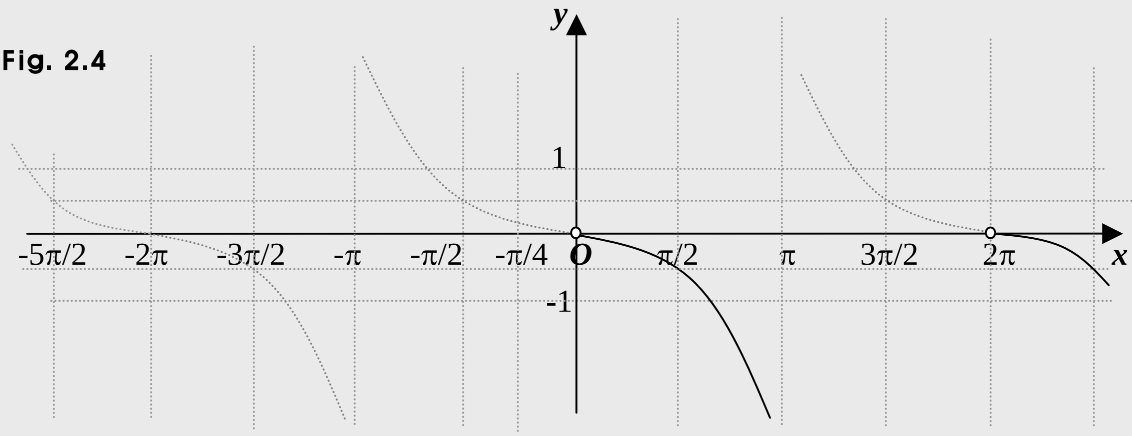


Next, we can put in a graph, the curve of $y = -\frac{1}{2}\tan\frac{x}{2}$ the way below.



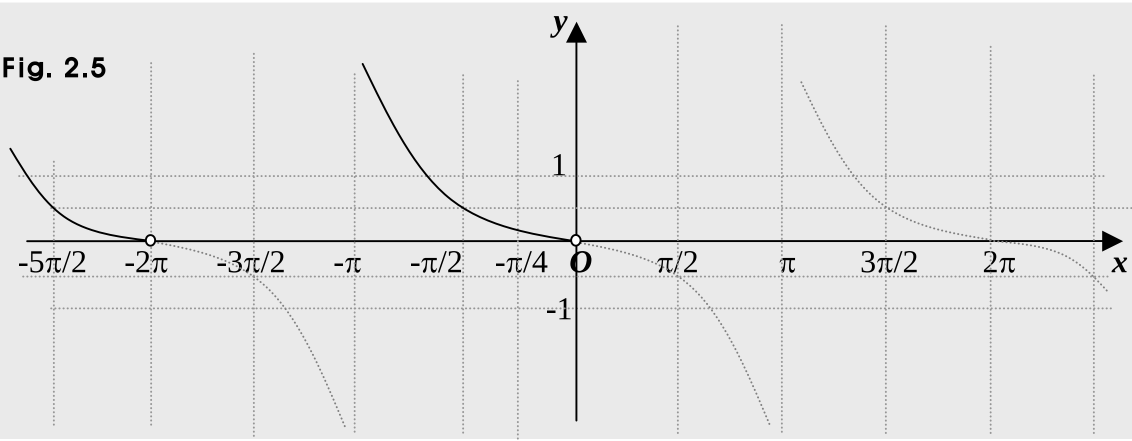
So next, moving on to the curve of $y = -\frac{1}{2}\tan\frac{x}{2}$ for $x \geq 0$, and $y < 0$, we can get this:

Fig. 2.4



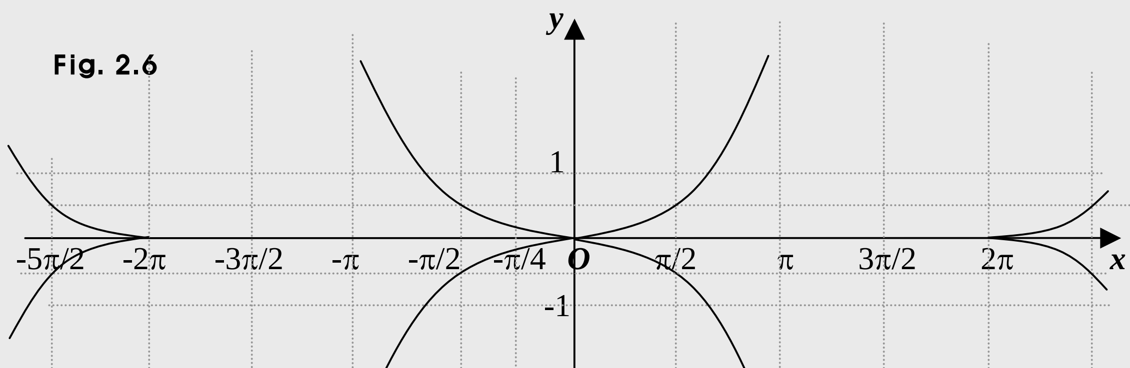
And next, moving on to the curve of $y = -\frac{1}{2}\tan\frac{x}{2}$ for $x < 0$, and $y \geq 0$, we can get this:

Fig. 2.5



Thus, putting in a graph, the curve of $|y| = \frac{1}{2}\tan\left|\frac{x}{2}\right|$, we can put it the way as follows.

Fig. 2.6



Notice that the curve is symmetric about the origin, as well as the x and y axes.
In fact, the curve of $|y| = f(|x|)$ is symmetric about the origin, as well as both the axes.

Suggestions or Solutions
To the Problem in the Example 3

Put in a graph the equations as follows: $\boldsymbol{y = \frac{1}{2}(\tan\frac{x}{2} + \tan|\frac{x}{2}|)}$.

To begin with, we can put the equation given the way below.
Assuming first, $\boldsymbol{x \geq 0}$, we get $\boldsymbol{y = \frac{1}{2}(\tan\frac{x}{2} + \tan\frac{x}{2}) = \tan\frac{x}{2}}$. So we get $\boldsymbol{y = \tan\frac{x}{2}}$ for $\boldsymbol{x \geq 0}$.

Assuming next, $\boldsymbol{x < 0}$, we get $\boldsymbol{y = \frac{1}{2}(\tan\frac{x}{2} - \tan\frac{x}{2}) = 0}$. So we get $\boldsymbol{y = 0}$ for $\boldsymbol{x < 0}$.

Thus next, putting in a graph, the curve of $\boldsymbol{y = \tan\frac{x}{2}}$, we can put it the way below:

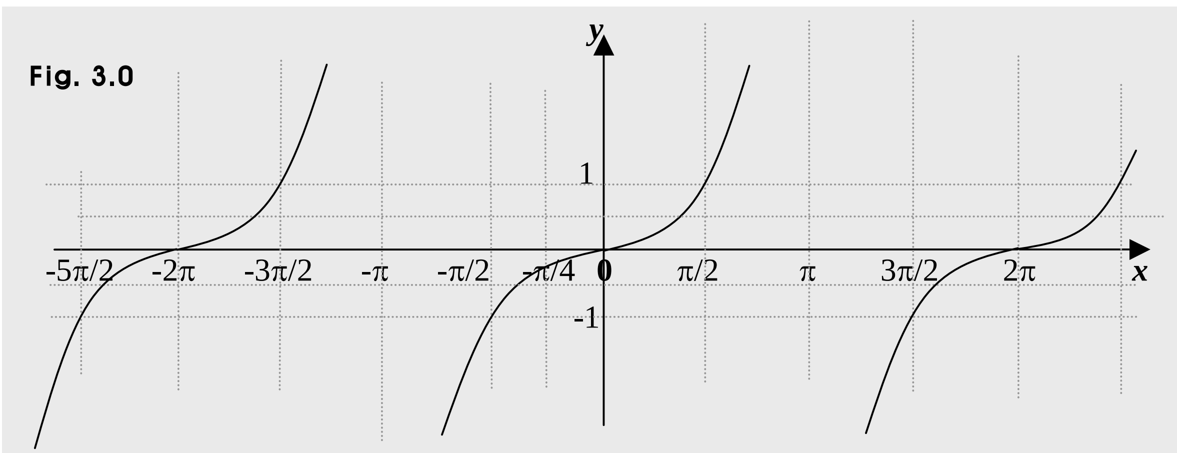


Fig. 3.0

Notice that when $\boldsymbol{x = 2n\pi + \pi/2}$ for $\boldsymbol{n}$ integer, we get $\boldsymbol{y = 1}$.
That is, we get $\boldsymbol{\tan (4n + 1)\pi/2 = 1}$ for $\boldsymbol{n}$ integer.

So putting in a graph, the curve of $\boldsymbol{y = \frac{1}{2}(\tan\frac{x}{2} + \tan|\frac{x}{2}|)}$, we can put it the way below.

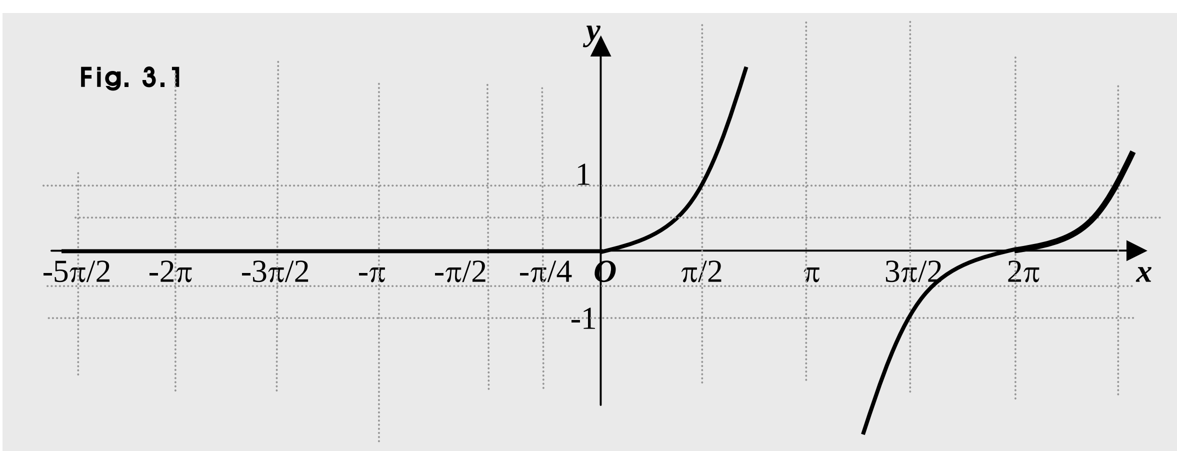
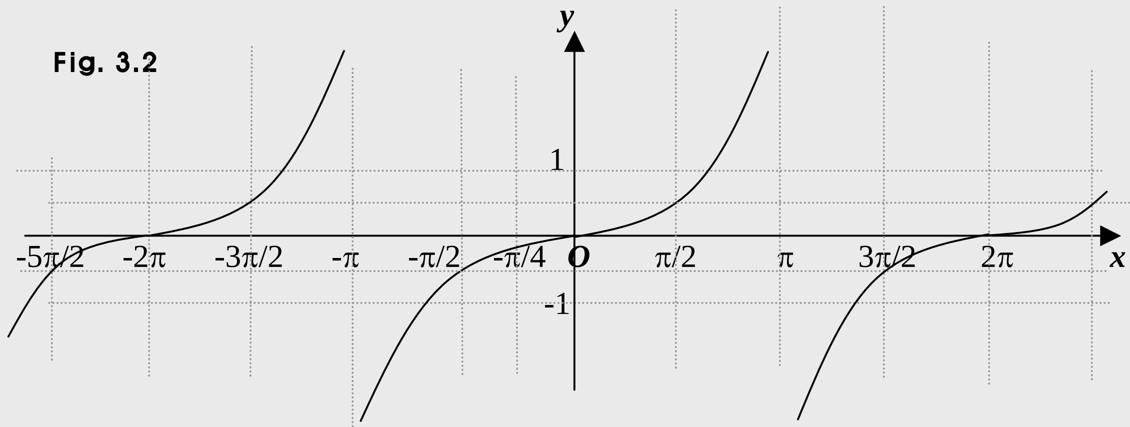


Fig. 3.1

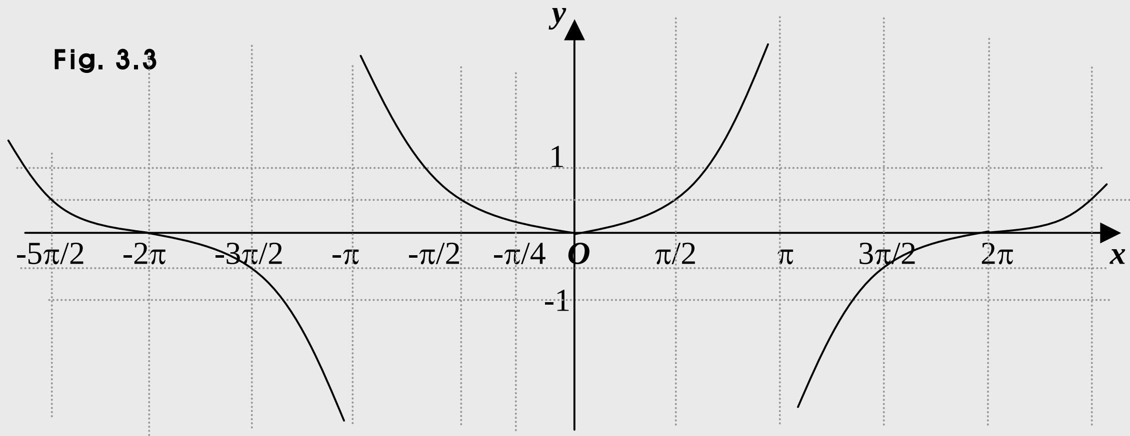
And of course, we can get the same using graphs. So to begin with, putting in a graph, the curve of $\boldsymbol{y = \frac{1}{2}\tan\frac{x}{2}}$, we can put it the way as follows.

Fig. 3.2



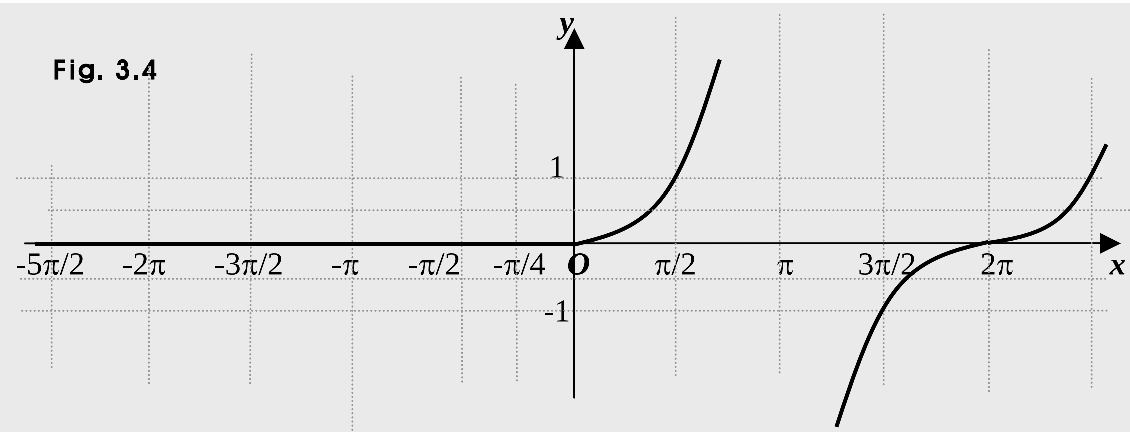
Next, putting in a graph, the curve of $\boldsymbol{y = \frac{1}{2}\tan\left|\frac{x}{2}\right|}$, we can put it the way below.

Fig. 3.3



Thus, adding together the two curves above, we can get this:

Fig. 3.4



Suggestions or Solutions
To the Problem in the Example 4

Put in a graph the equations as follows: $|y| = \frac{1}{2}(\tan\frac{x}{2} + \tan|\frac{x}{2}|)$.

To begin with, we know $\tan|\frac{x}{2}| = \tan\frac{1}{2}|x|$.

And we know $|y| \geq 0$ for all y, and $|x| \geq 0$ for all x.

So we want to consider four different cases as follows.

First, assuming $x \geq 0$, and $y \geq 0$, we get $y = \tan\frac{x}{2}$.

Next, assuming $x \geq 0$, and $y < 0$, we get $-y = \tan\frac{x}{2}$, so we get $y = -\tan\frac{x}{2}$.

Next, assuming $x < 0$, and $y < 0$, we get $-y = \frac{1}{2}\{\tan\frac{x}{2} + \tan(-\frac{x}{2})\} = 0$, so we get $y = 0$, which is not allowed, because we assume that $x < 0$, and $y < 0$.

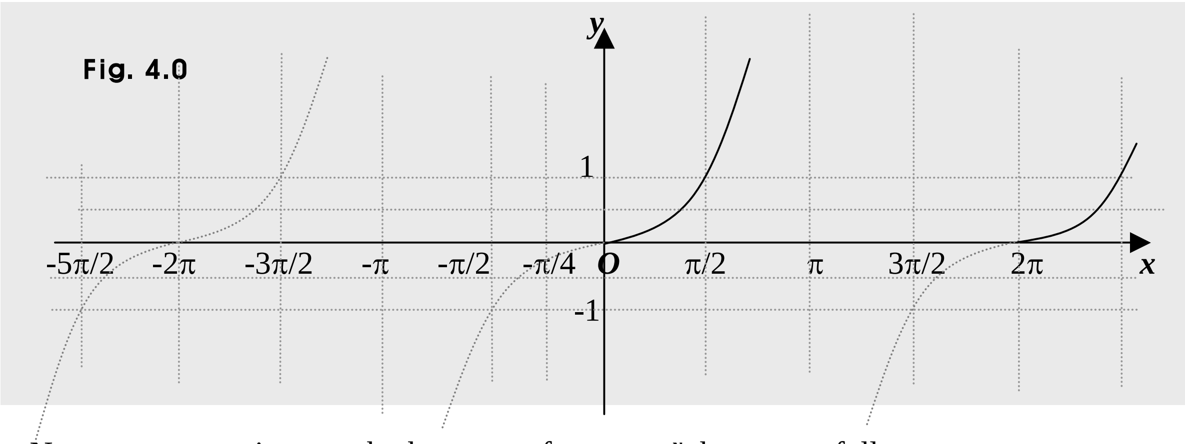
And next, assuming $x < 0$, and $y \geq 0$, we get $y = \frac{1}{2}\{\tan\frac{x}{2} + \tan(-\frac{x}{2})\} = 0$, so we get $y = 0$.

Thus in sum, we have these:

$y = \tan\frac{x}{2}$ for $x \geq 0$, and $y \geq 0$.	none for $x < 0$, and $y < 0$.
$y = -\tan\frac{x}{2}$ for $x \geq 0$, and $y < 0$.	$y = 0$ for $x < 0$, and $y \geq 0$.

So beginning with the curve of $y = \tan\frac{x}{2}$ for $x \geq 0$, and $y \geq 0$, we can get this:

Fig. 4.0



Next, we can put in a graph, the curve of $y = -\tan\frac{x}{2}$ the way as follows.

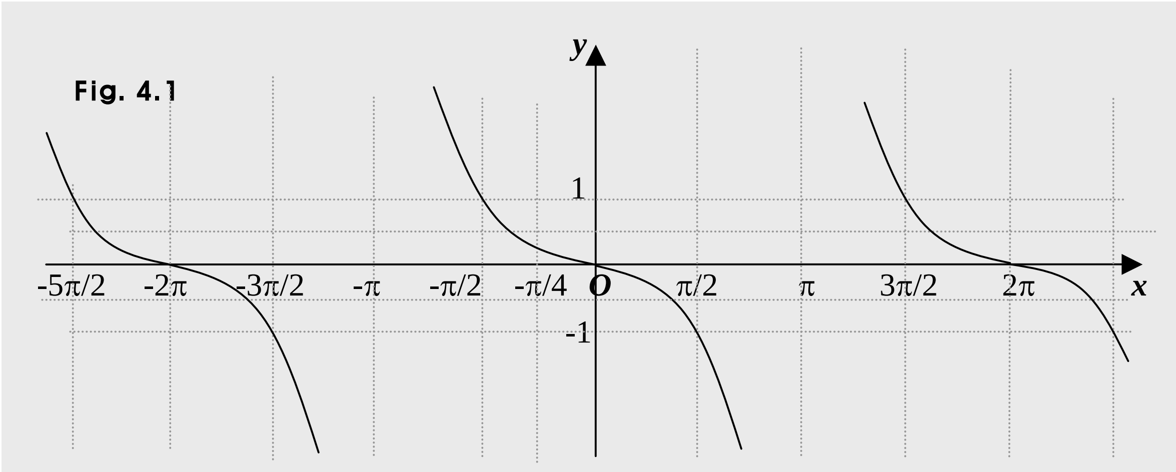


Fig. 4.1

So next, moving on to the curve of $\boldsymbol{y = -\tan\frac{x}{2}}$ for $\boldsymbol{x \geq 0}$, and $\boldsymbol{y < 0}$, we can get this:

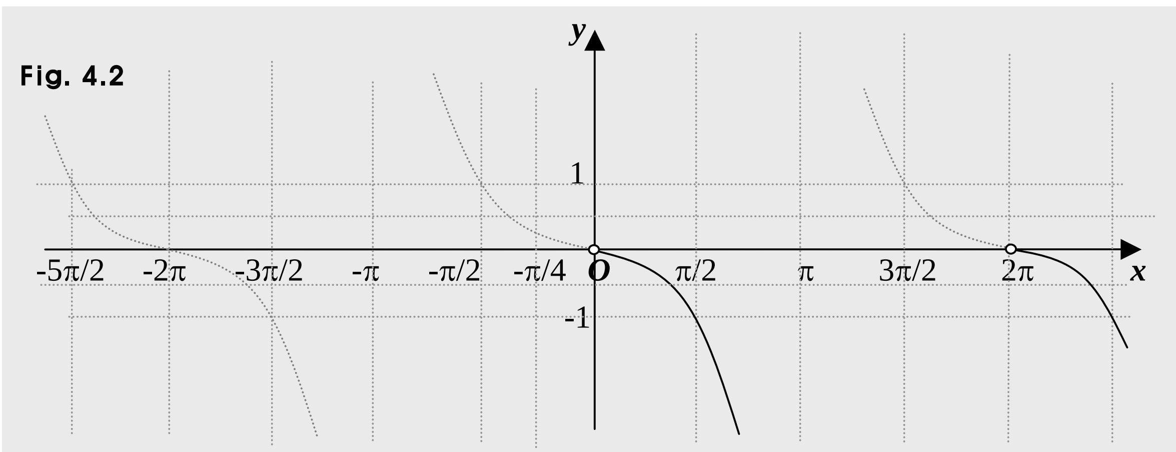


Fig. 4.2

And we have $\boldsymbol{y = 0}$ for $\boldsymbol{x < 0}$, and $\boldsymbol{y \geq 0}$.

And thus, we can put in a graph, the curve of $|y| = \frac{1}{2}(\tan\frac{x}{2} + \tan|\frac{x}{2}|)$ the way below:

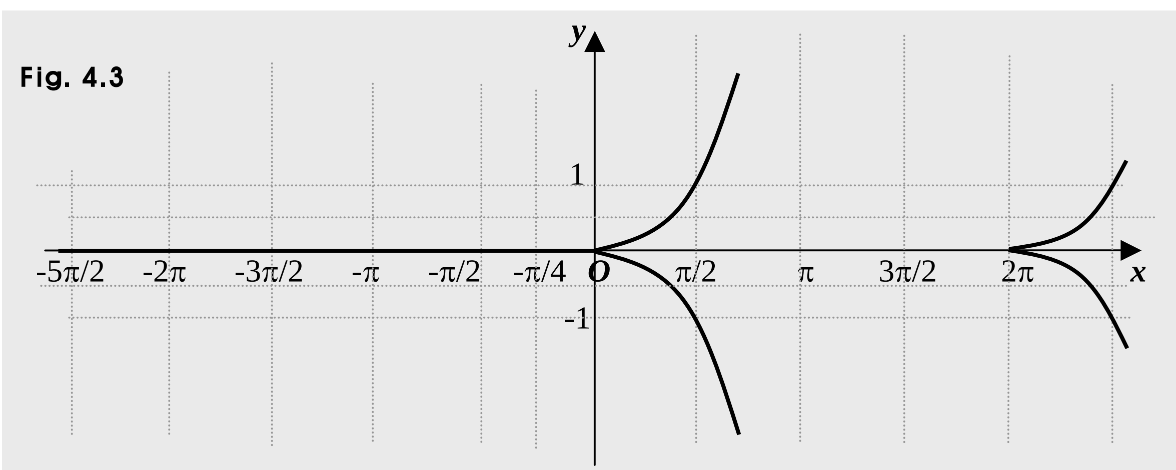


Fig. 4.3