Examples 1 in The Radix

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Examples 1 in The Radix

Take the sum of and the difference between the two numbers in parentheses. Note that the subscript inside the square brackets is the base used in their number system.

Example: $(25, 16)_{[7]}$
The subscript 7 in [ ] indicates that 25 and 16 are on the base 7. Therefore, The sum of 25 and 16 is 25 + 16 = 44. The difference between 25 and 16 is 25 – 16 = 6.

0. $(23, 12)_{[4]}$ **1.** $(110, 101)_{[2]}$ **2.** $(43, 24)_{[5]}$

3. $(43, 35)_{[6]}$ **4**. $(87, 65)_{[9]}$ **5.** $(10101, 111)_{[2]}$

6. $(52, 37)_{[8]}$ **7.** $(11011, 10101)_{[2]}$ **8.** $(53, 26)_{[7]}$

9. $(BAAB, 2A4B)_{[12]}$ **A.** $(A8B9, 7BAB)_{[14]}$

B. $(CCAD, 96CB)_{[16]}$ **C.** $(9DAG, 8FG3)_{[17]}$

D. $(HD32K, ABG7L)_{[24]}$

Suggestions or Solutions

To the Problem 0

$(23, 12)_{[4]}$

The numbers are on the base 4, so a unit for each digit has 4 equal pieces. And on the base 4, the sequence of place values proceeds the way as follows.

$\ldots, 4^6, 4^5, 4^4, 4^3, 4^2, 4^1, 4^0, 4^{-1}, 4^{-2}, 4^{-3}, 4^{-4}, \ldots$

So putting $23_{[4]}$ in decimal, we get

$23_{[4]} = 2 \times 4^1 + 3 \times 4^0 = 2 \times 4 + 3 \times 1 = 11$

We use a subscript to specify the base. Normally though, we omit the subscript in decimal system. And for simplicity, we can omit the square brackets if not ambiguous. For instance, we can put $83_{[8]}$ in just 83_8 if not confusing.

Let's now begin with the addition first.

Then, we get $23_4 + 12_4 = 101_4$. How?

In 23_4, the 1's digit shows 3 of 1s, and in 12_4, the 1's digit shows 2 of 1s. So we have 5 of 1's for 1's digit in sum, and have to make a carryover, because of the base 4.
Thus, we get $3_4 + 2_4 = 5 = 1 \times 4 + 1 = 11_4$.

Now we want to see what happens in the 4's digit.

In 23_4, the 4's digit shows 2 of 4s, and in 12_4, the 4's digit shows 1 of 4s. So we have 3 of 4s for 4's digit in sum, and yet, we have a carryover from the 1's digit.

Thus, the 4's place will get 4 of 4s in total, but 4 cannot be used, since the radix is 4, so we have to make a carryover to ($4^2 = 4 \times 4 = 16$)'s digit.

So the ($4^2 = 4 \times 4 = 16$)'s digit gets 1 of 4^2s in decimal, that is, one of 16s in decimal, which is 100_4.

Now, we have 1 in the 1's digit, 0 in the 4's digit, and 1 in the ($4^2 = 16$)'s digit.
Therefore, $(23 + 12)_4 = (1 \times 100 + 0 \times 10 + 1 \times 1)_4 = 101_4$.

Of course, we can convert the numbers on the base 4 to the numbers in decimal, find the sum, and then, convert the sum back to the number on the base 4.

Then first, we get $(23 + 12)_4$

$= (2 \times 4 + 3 \times 1 + 1 \times 4 + 2 \times 1)_{10} = (8 + 3 + 4 + 2)_{10} = 17_{10}$

Next, we want to convert the sum on the base 10 back to the number on the base 4 the way as follows.

We often omit the subscript in decimal system.

So we get

$17 = 16 + 1 = 1 \times 4^2 + 0 \times 4 + 1 = 101_4$

How do we do the operation in general though?

We do additions digit by digit in general.

We have $23_4 = 2 \times 4 + 3 \times 1$, and $12_4 = 1 \times 4 + 2 \times 1$.

So we get

$23_4 + 12_4 = (2 + 1) \times 4 + (3 + 2) \times 1 = 3 \times 4 + 5 \times 1$

$= 3 \times 4 + 1 \times 4 + 1 = (3 + 1) \times 4 + 1 = 4 \times 4 + 1$

$= 1 \times 4 \times 4 + 0 \times 4 + 1 = 1 \times 4^2 + 0 \times 4^1 + 1 \times 4^0 = 101_4$.

Therefore, $23_4 + 12_4 = 101_4$.

Note that

$1 \times 10 = 10^1 = 10$, but $10_4 = 1 \times 4^1 = 4$

$1 \times 100 = 1 \times 10^2 = 100$, but $100_4 = 1 \times 4^2 = 16$

$1 \times 4 \times 4 = 1 \times 4^2 = 100$ on the base 4, since 100_4 has 1 of ($4 \times 4 = 4^2 = 16$)s, but does not have any of 4s or 1s.

Let's now take the **difference** between 23_4 and 12_4.

$23_4 - 12_4 = 11_4$. How?

By convention, we do a subtraction digit by digit, and normally, we begin with the lowest digit.

For 1's digit, we get $3 - 2 = 1$.
For 4's digit, we get $2 - 1 = 1$.
So the difference = 11_4, which means $1 \times 4 + 1 = 11_4$.

Of course, we can convert the numbers on the base 4 to the numbers on the base 10, do the operation, and then, convert the result back to the number on the base 4.

Suggestions or Solutions

To the Problem 1

$(110, 101)_{[2]}$

On the base 2, each unit is a set of 2 equal pieces.
Beginning with the addition first, we get

$110_2 + 101_2 = 1011_2$.

By convention, we do an addition digit by digit, and normally, begin with the lowest digit. When doing it digit by digit, we do it on corresponding digits, of course.

For 1's digit, we get $0 + 1 = 1$
For 2's digit, we get $1 + 0 = 1$
For $2^2 = 4$'s digit, we get $1 + 1 = 2$. We know $2 = 10_2$.
So for the 4's digit, we get 0, but we have to increase 1 for the next higher digit, which is $2^3 = 8$'s digit, which is 0 at the beginning. So we have a carryover to the 8's digit which is 0. Thus, we'll have 1 for the 8's digit.

So we get the sum as follows. $110_2 + 101_2 = 1011_2$.
Of course, we can convert the numbers to decimal equivalents, do the addition, and convert the sum on the base 10 back to the number on the base 2.

$110_2 + 101_2 = 1 \times 4 + 1 \times 2 + 0 \times 1 + 1 \times 4 + 0 \times 2 + 1 \times 1$
$= (1 \times 4 + 1 \times 4) + (1 \times 2 + 0 \times 2) + (0 \times 1 + 1 \times 1)$
$= (1 + 1) \times 4 + (1 + 0) \times 2 + (0 + 1) \times 1$
$= 2 \times 4 + 1 \times 2 + 1 \times 1$
$= 2 \times 2 \times 2 + 1 \times 2 + 1 \times 1$
$= \mathbf{1} \times 2 \times 2 \times 2 + \mathbf{0} \times 2 \times 2 + \mathbf{1} \times 2 + \mathbf{1} \times 1$
$= \mathbf{1} \times 2^3 + \mathbf{0} \times 2^2 + \mathbf{1} \times 2^1 + \mathbf{1} \times 2^0$.
$= \mathbf{1011}_2$.

Notice that 1011_2 has none of ($2 \times 2 = 2^2$)s.
Let's see now what happens with the difference.

$110_2 – 101_2 = 1_2$.   Note that in every number system, 0 and 1 are 0 and 1.
By convention, we do a subtraction digit by digit, and normally, begin with the lowest digit.

For 1's digit, we have to subtract 1 from 0, but we can’t, so we need to take 1 unit from the next higher digit in 110.

We know $110_2 = 100_2 + 10_2 = 100_2 + 1 \times 2$.

So we get $110_2 – 101_2 = 100_2 + 1 \times 2 – (100_2 + 1)$.

$= 100_2 – 100_2 + 1 \times 2 – 1 = 2 –1 = 1$.

Suggestions or Solutions

To the Problem 2

(43, 24)$_{[5]}$

$43_5 + 24_5 = 122_5$ $43_5 – 24_5 = 14_5$

The numbers are on the base 5, so each units has 5 equal pieces. Thus, each digit can show from 0 up to 4 units, that is, each place can get from 0 up to 4 units. In other words, each digit can be one of 0, 1, 2, 3, and 4 only.

Doing the arithmetic operations, we can break the numbers apart into digits, and then, do the operations digit by digit.

43 = 40 + 3

24 = 20 + 4

43 + 24 = 60 + 7 = **50** + 10 + **5** + 2 = **100** + 10 + **10** + 2

= 100 + 20 + 2 = 122

Why are 6 and 5 used though?

Of course, we are not allowed to use 5 or 6 on the base 5, but the use of such a number is just for efficiency purposes in the operations. It's easy to keep track of and control calculation flow using those numbers. For instance, it is illegal to use 50 in 5-number system, but 50 means 100_5 in such a way as follows.

$50_5 = 5 \times 10_5 = (1 \times 10 \times 10)_5 = 100_5$.

And 60 means 6 units, each of which is worth 5 in decimal. Note that it is for **efficiency purposes only**, and that it is **not** a theory or any rule or principle at all.

If each digit is more than 4, we need to make a carryover to the next higher digit, since the numbers are on the base 5.

$43 = 40 + 3 = 30 + 10 + 3 = \mathbf{30 + 5 + 3}$

$24 = \mathbf{20 + 4}$

$43 – 24 = (\mathbf{30 – 20}) \mathbf{+} (\mathbf{5 – 4}) \mathbf{+ 3} = 10 + 1 + 3 = 10 + 4 = 14$

If a particular digit to be subtracted from is smaller,
we need to take one unit from the next higher digit,
do the subtraction, add the difference to the particular digit,
and then, take the sum for the result for the particular digit.

Suggestions or Solutions

To the Problem 3

(43, 35)$_{[6]}$

Each digit in a number on the base 6 can be from 0 up to 5. And each unit has 6 equal pieces.

$43_6 + 35_6 = 122_6$. $43_6 - 35_6 = 4_6$.

We can break numbers apart into digits, and proceed with the operation digit by digit. And doing the operations, keep the radix 6 in your mind. So if you see 6 doing the operation, switch it to 10.

43 = 40 + 3

35 = 30 + 5

---

43 + 35 = 40 + 30 + 3 + 5 = 70 + 8 = **60** + 10 + 6 + 2

= **100** + 10 + 10 + 2 = **100** + 20 + 2 = 122

Note that $10_6 = 6_{10}$, and $20_6 = 2 \times 6_{10}$.

Of course, we are not allowed to use 6 in a number on the base 6, but the use of the 6 is just for efficiency purpose in the operations. For instance, it is illegal to use 60 on the base 6, but 60_6 means 100_6 in such a way as follows.

$60_6 = 6 \times 10_6 = (1 \times 10 \times 10)_6 = 100_6$

$43 = 40 + 3 = 30 + 10 + 3 = \mathbf{30 + 6 + 3}$

$35 = \mathbf{30 + 5}$

$43 - 35 = \mathbf{30 - 30 + 1 + 3} = 00 + 4 = 4$

Note that 00 is for efficiency purposes only.

Suggestions or Solutions

To the Problem 4

(87, 65)[9] $87_9 + 65_9 = 163_9$ $87_9 - 65_9 = 22_9$

Each place in a number on the base 9 can get from 0 up to 8 units, each of which has 9 equal pieces.

87 = 80 + 7

65 = 60 + 5

87 + 65 = 80 + 60 + 7 + 5 = 140 + 12 = **90** + 50 + 9 + 3

= **100** + 50 + 10 + 3 = 163

Of course, we are not allowed to use 9 on the base 9, but the use of the 9 is just for efficiency purpose. For instance, it is illegal to use 90 in 9-number system, but 90_9 means 100_9 the way as follows. $90_9 = 9 \times 10_9 = (1 \times 10 \times 10)_9 = 100_9$.

87 = 80 + 7

65 = 60 + 5

87 – 65 = 20 + 2 = 22

Suggestions or Solutions
To the Problem 5

(10101, 111)[2]

$10101_2 + 111_2 = 11100_2$ $\qquad$ $10101_2 - 111_2 = 1110_2$

10101 = 10000 + 100 + 1

111 =100 + 10 + 1

10101 + 111 = 10000 + 200 + 10 + **2**

= 10000 + 1000 + 10 + **10** = 11000 + 20

= 11000 + 100 = 11100

Of course, we are not allowed to use 2 on the base 2, but the use of the 2 is just for efficiency purposes. For instance, it is illegal to use 20 in binary system, but 20_2 means 100_2 in such a way as follows.

$20_2 = 2 \times 10_2 = (1 \times 10 \times 10)_2 = 100_2$. By the same token, 200_2 means 1000_2.

10101 = 10000 + **100** + 1 = 10000 + **20** + 1

= 2000 + **20** + 1 = 1000 + **1000** + **20** + 1

= 1000 + **200** + **20** + 1

111 = 100 + 10 + 1

10101 – 111 = 1000 + 200 – 100 + **20** – 10 + 1 – 1

= 1000 + 100 + 10 + 0 = 1110

Note that $200_2 = 1000_2$, and $20_2 = 100_2$.

The use of 2 the way above is for efficiency purposes only.

Suggestions or Solutions

To the Problem 6

$(52, 37)_{[8]}$

$52_8 + 37_8 = 111_8$ $52_8 - 37_8 = 13_8$

$52 = 50 + 2$

$\underline{37 = 30 + 7}$

$52 + 37 = 80 + 9 = 100 + 8 + 1 = 100 + 10 + 1 = 111$

Of course, we are not allowed to use 8 on the base 8, but the use of 8 is just for efficiency purposes. For instance, it is illegal to use 8 in binary system, but 8_8 means 10_8 in such a way as follows. $8_8 = 8 \times 1_8 = (10 \times 1)_8 = 10_8$.
By the same token, 80_2 means $8_8 \times 10_8 = 10_8 \times 10_8 = 100_8$.
Note that it is for efficiency purposes only.

$52 = 50 + 2 = \mathbf{40 + 8 + 2}$

$\underline{37 = \mathbf{30 + 7}}$

$52 - 37 = \mathbf{10} + \mathbf{1} + \mathbf{2} = 13$

Suggestions or Solutions

To the Problem 7

$(11011, 10101)_{[2]}$

$11011_2 + 10101_2 = 110000_2$ $11011_2 - 10101_2 = 110_2$

The numbers are binary, so each unit is made of 2 equal pieces, and each place takes from 0 up to 1 unit, and thus, each digit can be one of 0 and 1 only.

11011 = 10000 + 1000 + 10 + 1

10101 = 10000 + 100 + 1

11011 + 10101 = 20000 + 1000 + 100 + 10 + **2**

= 100000 + 1000 + 100 + 10 + **10**

= 100000 + 1000 + 100 + 20

= 100000 + 1000 + 100 + 100

= 100000 + 1000 + 200

= 100000 + 1000 + 1000

= 100000 + 2000

= 100000 + 10000

= 110000

11011 = 10000 + 1000 + 10 + 1 = **10000 + 200 + 10 + 1**

10101 = **10000 + 100 + 1**

11011 – 10101 =10000 – 10000 + 200 – 100 + 10 + 1 – 1

= 100 + 10 = 110

Suggestions or Solutions

To the Problem 8

(53, 26)[7]

$53_7 + 26_7 = 112_7$ $53_7 – 26_7 = 24_7$

The numbers are on the base 7, so each unit is made of 7 equal pieces, and each place takes from 0 up to 6 units, and thus, each digit can be one of 0, 1, 2, 3, 4, 5, and 6 only.

53 = 50 + 3

26 = 20 + 6

53 + 26 = 70 + 9 = 100 + 7 + 2 = 100 + 10 + 2 = 112

53 = 50 + 3 = 40 + 10 + 3 = **40 + 7 + 3**

26 = **20 + 6**

53 - 26 = 20 + 1 + 3 = 24

Suggestions or Solutions

To the Problem 9

$(BAAB, 2A4B)_{[12]}$

$A_{12} = 10$, and $B_{12} = 11$

$BAAB_{12} + 2A4B_{12} = 1293A_{12}$ $\quad$ $BAAB_{12} - 2A4B_{12} = 9060_{12}$

BAAB = B000 + A00 + A0 + B

2A4B = 2000 + A00 + 40 + B

BAAB + 2A4B = (B + 2)000 + (A + A)00 + (A + 4)0 + B + B

= (11 + 2)000 + (10 + 10)00 + (10 + 4)0 + (11 + 11)

= (13)000 + (20)00 + (14)0 + (22)

= (12 + 1)000 + (12 + 8)00 + (12 + 2)0 + (12) + A

= (12)000 + 1000 + (12)00 + 800 + (12)0 + 20 + 10 + A

= 10000 + 1000 + 1000 + 800 + 100 + 30 + A

= 10000 + 2000 + 900 + 30 + A

= 1293A

We are using another trick as follows.
We can use 0s to simplify the expression of a number on a particular base the way as follows.

(12)000 on the base 17 can be put this way:

$12_{[10]}$ x $1000_{[17]}$ = $C000_{[17]}$

(25)00 on the base 17 can be put this way:

($17_{[10]}$ + $8_{[10]}$) x $100_{[17]}$ = ($10_{[17]}$ + $8_{[17]}$) x $100_{[17]}$

= $18_{[17]}$ x $100_{[17]}$ = $1800_{[17]}$}

Clearly, it is illegal to use (B + 2)000 in a number on any base. However, the usage is for efficiency purposes only. (B + 2)000 means that the fourth digit (not a fourth's digit) would have to hold B + 2 of its units. What is the unit?

It is 12 x 12 x 12 = 12^3, and is the place value. So we mean that (B + 2)000 has B + 2 of 12^3s. We can use this trick for fast computations, since we are used to decimal system.

However, we need to be careful with the usage admitting the fact that it is not legal so that we cannot use it in public, and use it for interim calculations only. The main idea in a number system is the units and their pieces, and it is important to keep up with the idea doing math operations with radices.

BAAB = B000 + A00 + A0 + B

2A4B = 2000 + A00 + 40 + B

BAAB – 2A4B = (B – 2)000 + (A – A)00 + (A – 4)0 + B – B

= (11 – 2)000 + 000 + (10 – 4)0 + 0

= 9000 + 60

= 9060

Suggestions or Solutions

To the Problem A

(A8B9, 7BAB)$_{[14]}$

A = 10, B = 11, C = 12, and D = 13

$A8B9_{14} + 7BAB_{14} = 14686_{14}$ $A8B9_{14} - 7BAB_{14} = 2B0C_{14}$

A8B9 = A000 + 800 + B0 + 9

7BAB = 7000 + B00 + A0 + B

A8B9 + 7BAB

= (10 + 7)000 + (11 + 8)00 + (11 + 10)0 + 9 + 11

= (17)000 + (19)00 + (21)0 + (20)

= (14 + 3)000 + (14 + 5)00 + (14 + 7)0 + (14 + 6)

= 10000 + 3000 + 1000 + 500 + 100 + 70 + 10 + 6

= 10000 + 4000 + 600 + 80 + 6

= 14686

In this problem too, we are now using the same tricks as the ones in the problem 9. Clearly, it is illegal to use a number like (10 + 7)000 on any base.

However, the usage is for efficiency purpose only.
(10 + 7)000 means that the fourth digit (not a fourth's digit) would have to hold 17 of its units. What then is the unit?

It is 14 x 14 x 14 = 14^3. Thus, we mean that (10 + 7)000 has 17 of 14^3s. We can use the trick for fast computations, since we are used to decimal system. However, we need to be careful with the usage admitting the fact that it is not legal so that we cannot use it in public, and use it for interim calculations only. The main idea in number systems is the units and their pieces, and it is important to keep up with the idea when working with numbers with difference radices.

Now, we are on the base 14 at the moment.

A8B9 = A000 + 800 + B0 + 9
= **9000** + (14 + 8)00 + A0 + (14 + 9)
7BAB = **7000** + B00 + A0 + B

A8B9 – 7BAB
= **2000** + (14 + 8 – 11)00 + (A – A)0 + (14 + 9 – 11)
= 2000 + B00 + 00 + C = 2B0C

Suggestions or Solutions
To the Problem B

(CCAD, 96CB)$_{[16]}$

A = 10, B = 11, C = 12, D = 13, E = 14, and F = 15

$CCAD_{16} + 96CB_{16} = 16378_{16}$ $CCAD_{16} - 96CB_{16} = 35E2_{16}$

On the base 16, each unit is a set of 16 equal pieces.

CCAD = C000 + C00 + A0 + D

96CB = 9000 + 600 + C0 + B

CCAD + 96CB

= (12 + 9)000 + (12 + 6)00 + (10 + 12)0 + (13 + 11)

= (21)000 + (18)00 + (22)0 + (24)

= (16 + 5)000 + (16 + 2)00 + (16 + 6)0 + (16 + 8)

= 10000 + 5000 + 1000 + 200 + 100 + 60 + 10 + 8

= 10000 + 6000 + 300 + 70 + 8

= 16378

CCAD = C000 + C00 + A0 + D

= (12)000 + B00 + (16 + 10)0 + D,

since C00 = B00 + (16)0 = B00 + 100, since 16 = 10_{16}

96CB = 9000 + 600 + C0 + B

CCAD – 96CB

= (12 – 9)000 + (11 – 6)00 + (16 + 10 – 12)0 + (13 – 11)

= 3000 + 500 + (14)0 + 2

= 3000 + 500 + E0 + 2 = 35E2

Suggestions or Solutions

To the Problem C

$(9DAG, 8FG3)_{[17]}$

A = 10, B = 11, C = 12, D = 13, E = 14, F = 15, and G = 16

$9DAG_{17} + 8FG3_{17} = 11CA2_{17}$ $9DAG_{17} – 8FG3_{17} = EBD_{17}$

9DAG = **9000** + D00 + A0 + G

8FG3 = **8000** + F00 + G0 + 3

9DAG + 8FG3

= **(17)000** + (13 + 15)00 + (10 + 16)0 + (16 + 3)

= **10000** + (17 + 11)00 + (17 + 9)0 + (17 + 2) $\{17_{10} = 10_{17}\}$

= 10000 + 1000 + (11)00 + 100 + 90 + 10 + 2

= 10000 + 1000 + B00 + 100 + (9 + 1)0 + 2

= 10000 + 1000 + C00 + A0 + 2

= 11CA2

9DAG = 9000 + D00 + A0 + G

= 8000 + (17 + 13)00 + A0 + (16)

= 8000 + (17 + 12)00 + (17 + 10)0 + (16)

8FG3 = 8000 + F00 + G0 + 3

9DAG – 8FG3

= (17 + 12 – 15)00 + (17 + 10 – 16)0 + (16 – 3),

since G = 16

= (14)00 + (11)0 + (13)

= EBD

Suggestions or Solutions

To the Problem D

(HD32K, ABG7L)$_{[24]}$

A = 10, B = 11, C = 12, D = 13, E = 14, F = 15, G = 16,
H = 17, I = 18, J = 19, K = 20, L = 21, M = 22, and N = 23

$HD32K_{24} + ABG7L_{24} = 140JAH_{24}$
$HD32K_{24} – ABG7L_{24} = 71AIN_{24}$

HD32K = H0000 + D000 + 300 + 20 + K
ABG7L = A0000 + B000 + G00 + 70 + L

HD32K + ABG7L
= (17 + 10)0000 + (13 + 11)000
+ (3 + 16)00 + (9)0 + (20 + 21)
= (27)0000 + (24)000 + (19)00 + (9)0 + (41)
= (24 + 3)0000 + 10000 + J00 + 90 + (24 + 17)
= 100000 + 30000 + 10000 + J00 + 90 + 10 + H
= 100000 + 40000 + J00 + A0 + H
= 140JAH

$$\begin{aligned}
HD32K &= H0000 + D000 + 300 + 20 + K \\
&= H0000 + C000 + (24 + 3)00 + 20 + K \\
&= H0000 + C000 + (24 + 2)00 + (24 + 2)0 + K \\
&= H0000 + C000 + (24 + 2)00 + (24 + 1)0 + (24 + 20)
\end{aligned}$$

$$\underline{ABG7L = A0000 + B000 + G00 + 70 + L}$$

$$\begin{aligned}
&HD32K - ABG7L \\
&= (H - A)0000 + 1000 + (24 + 2 - 16)00 \\
&+ (24 + 1 - 7)0 + (24 + 20 - 21) \\
&= 70000 + 1000 + (10)00 + (18)0 + (23) \\
&= 70000 + 1000 + A00 + I0 + N \\
&= 71AIN
\end{aligned}$$