Examples 3 in The Radix

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Convert the number on the base indicated inside the square brackets into the decimal equivalent.

Examples: $10_{[3]} = 3_{[10]} = 3$, $21_{[4]} = 9$, $32_{[5]} = 17$, $123_{[4]} = 27$,

$0.12_{[4]} = 0.375$, $12.32_{[5]} = 7.68$, and

$10.23_{[6]} = 6 + 15/36$, or $231/36$

0. $12.323_{[4]}$ **1**. $1.101_{[2]}$ **2**. $40.3_{[5]}$

3. $4.305_{[6]}$ **4**. $0.87_{[9]}$ **5**. $101.101_{[2]}$

6. $702.572_{[8]}$ **7**. $111.11_{[2]}$ **8**. $10.101_{[7]}$

9. $B3.A0B_{[12]}$ **10**. $A8.0BAC_{[14]}$ **11**. $CB1.A0D_{[16]}$

12. $9D0.ABG_{[17]}$ **13**. $H0D.3A2K_{[24]}$

Suggestions or Solutions
To the Problem 0

$12.323_{[4]}$

On the base B, which is an integer ≥ 2, each digit can be from 0 up to B – 1.
And on the base B, the sequence of place values proceeds the way as follows.
…, B^6, B^5, B^4, B^3, B^2, B^1, B^0, B^{-1}, B^{-2}, B^{-3}, B^{-4}, …

So the place value for the 1's digit is 1, the place value for the next higher digit is B, and we call the digit the B's digit.

Then, the next higher place value is B x B, and the next higher place value is B x B x B = B^3. Then, the next higher is B^4, and so forth. On the other hand, the next lower place than the 1's place has its place value of 1/B, which is B^{-1}.
So the digit there is called (1/B)'s digit or B^{-1}'s place.

The next lower place has its place value of (1/B) x (1/B) = $(1/B)^2 = 1/B^2 = B^{-2}$. Then, the next lower is $(1/B)^3 = B^{-3}$, and so forth.

When we convert a number on a particular base to the number in decimal, we use the place values in accordance with the particular base. Then, the decimal number has the same value as the number to be converted.

$12.323_{[4]}$ has 1 of 4s, 2 of 1s, 3 of (1/4)s, 2 of {1/(4x4)}s, and 3 of {1/(4x4x4)}s.
So we get
$12.323_{[4]}$ = 1x4 + 2x1 + 3 x 1/4 + 2 x 1/(4x4) + 3 x 1/(4x4x4)
= 4 + 2 + 3/4 + 2/(4x4) + 3/(4x4x4)
= 6 + 3/4 + 1/8 + 3/64
= 6 + (3x16 + 1x8 + 3)/64
= 6 + 59/64, which may be simplified further.
= $6.921875_{[10]}$.

Therefore, 12.323 on the base 4 is the same as 6 + 59/64 = 6.921875 on the base 10.

Suggestions or Solutions
To the Problem 1

$1.101_{[2]}$

$1.101_{[2]} = 1 + 1 \times 1/2 + 0 \times 1/(2x2) + 1 \times 1/(2x2x2)$

$= 1 + 1/2 + 0 + 1/8$

$= 1 + (4 + 1)/8$

$= 1 + 5/8$

$= 1.625$

We don’t usually use the subscript for a decimal number.

Suggestions or Solutions
To the Problem 2

$40.3_{[5]}$

40.3 on the base 5 has 4 of 5s, none of 1s, and 3 of (1/5)s.

$40.3_{[5]} = 4 \times 5 + 0 \times 1 + 3 \times 1/5$

$= 4 \times 5^{1} + 0 \times 5^{0} + 3 \times 5^{-1}$

$= 20 + 3/5$

$= 20.6$

Suggestions or Solutions
To the Problem 3

$4.305_{[6]}$

$4.305_{[6]} = 4 \times 1 + 3 \times 1/6 + 0 \times 1/(6x6) + 5 \times 1/(6x6x6)$

$= 4 + 3/6 + 0 + 5/(6x6x6)$

$= 4 + (3 \times 36 + 5)/216$

$= 4 + 113/216$

Suggestions or Solutions

To the Problem 4

$0.87_{[9]}$

$0.87_{[9]} = 5 + 8/9 + 7/(9x9)$

$= 5 + (72 + 7)/81$

$= 5 + 79/81$

Suggestions or Solutions

To the Problem 5

$101.101_{[2]}$

$101.101_{[2]} = 1 \times 2x2 + 0 \times 2 + 1 + 1/2 + 0/(2x2) + 1/(2x2x2)$

$= 4 + 1 + (4 + 1)/8$

$= 5 + 5/8$

$= 5.625$

Suggestions or Solutions

To the Problem 6

$702.572_{[8]}$

$702.572_{[8]} = 7 \times 8x8 + 0 \times 8 + 2 + 5/8 + 7/(8x8) + 2/(8x8x8)$

$= 448 + 2 + (5x64 + 7x8 + 2)/512$

$= 450 + (320 + 56 + 2)/512$

$= 450 + 378/512$

$= 450.73828125$

Suggestions or Solutions

To the Problem 7

$111.11_{[2]}$

$111.11_{[2]} = 1 \times 2x2 + 1 \times 2 + 1 + 1/2 + 1/(2x2)$

$= 4 + 2 + 1 + (2 + 1)/4$

$= 7 + 3/4$

$= 7.75$

Suggestions or Solutions

To the Problem 8

$10.101_{[7]}$

$10.101_{[7]} = 1 \times 7 + 0 + 1/7 + 0/(7x7) + 1/(7x7x7)$

$= 7 + (49 + 1)/343$

$= 7 + 50/343$

Suggestions or Solutions

To the Problem 9

$B3.A0B_{[12]}$

$B3.A0B_{[12]} = B \times 12 + 3 + A/12 + 0/(12x12) + B/(12x12x12)$

$= 11 \times 21 + 3 + 10/12 + 0 + 11/(12x12x12)$

$= 231 + 3 + (10 \times 12 \times 12 + 11)/(12x12x12)$

$= 234 + (1440 + 11)/1728$

$= 234 + 1451/1728$

Suggestions or Solutions
To the Problem A

$A8.0BAC_{[14]}$

It is useful to use exponents when we convert a non decimal number into a decimal number.

A8 is an integer on the base 14. So A8 on the base 14 is A x 14 + 8 on the base 10 where A = 11.

0.0BAC is a fraction on the base 14.
0.0BAC = 0.0B + 0.00A + 0.000C

0.0B on the base 14 = B x $1/14^2$ on the base 10
= $11/14^2$

0.00A on the base 14 = A x 1/(14 x 14 x 14) on the base 10
= $A/14^3$ on the base 10 = $10/14^3$ on the base 10

0.000C on the base 14

= C x /(14 x 14 x 14 x 14) on the base 10

= $C/14^4$ on the base 10

= $12/14^4$ on the base 10

Now, we put the integers and the fractions all together.

$A8.0BAC_{[14]}$

= $A \times 14 + 8 + 0/14 + B/14^2 + A/14^3 + C/14^4$

= $148 + (11 \times 14^2 + 10 \times 14 + 12)/14^4$

= 148 + (2156 + 140 + 12)/38416

= 148 + 2308/38416

= 148 + 577/9604

Suggestions or Solutions

To the Problem B

$CB1.A0D_{[16]}$

$CB1.A0D_{[16]}$

$= C \times 16x16 + B \times 16 + 1 + A/16 + 0/(16x16) + D/16^3$

$= 12 \times 16^2 + 11 \times 16 + 1 + (10 \times 16^2 + 12)/16^3$

$= 3072 + 176 + 1 + (2560 + 12)/4096$

$= 3249 + 2572/4096$

$= 3246 + 643/1024$

Suggestions or Solutions
To the Problem C

$9D0.ABG_{[17]}$

$9D0.ABG_{[17]}$

$= 9 \times 17^2 + D \times 17 + 0 + A/17 + B/17^2 + G/17^3$

$= 9 \times 289 + 13 \times 17 + (10 \times 17^2 + 11 \times 17 + 16)/17^3$

$= 2601 + 221 + (289 + 187 + 16)/4913$

$= 2822 + 492/4913$

Suggestions or Solutions
To the Problem D

$H0D.3A2K_{[24]}$

$H0D.3A2K_{[24]}$

$= H \times 24^2 + 0 \times 24 + D + 3/24 + A/24^2 + 2/24^3 + K/24^4$

$= 17 \times 576 + 0 + 13 + (3 \times 24^3 + A \times 24^2 + 2 \times 24 + 20)/24^4$

$= 9792 + 13 + (3 \times 13824 + 10 \times 576 + 48 + 20)/331776$

$= 9805 + (41472 + 5760 + 68)/331776$

$= 9805 + 47230/331776$

$=9805 + 23615/165888$