Examples 4 in The Radix

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Examples 4 in The Radix

Convert the decimal numbers into the numbers on the bases indicated inside the square brackets.

Examples:

3 → [3] $3 = 10_{[3]}$ 6 → [4] $6 = 12_{[4]}$

7 → [5] $7 = 12_{[5]}$ 37 → [6] $37 = 101_{[6]}$

0. 12 → [3] **1**. 19 → [4] **2**. 403 → [2]

3. 4305 → [5] **4**. 87 → [7] **5**. 101101 → [2]

6. 702572 → [8] **7**. 11111 → [9]

8. 10101 → [11] **9**. 3462654 → [12]

10. 752015 → [14] **11**. 20022403 → [16]

12. 82031724 → [17] **13**. 3054802 → [24]

Suggestions or Solutions
To the Problem 0

12 → [3]

On the base 3, each unit is a set of 3 equal pieces.
And on the base 3, the sequence of place values proceeds the way as follows.
…, 3^6, 3^5, 3^4, 3^3, 3^2, 3^1, 3^0, 3^{-1}, 3^{-2}, 3^{-3}, 3^{-4}, …

12
= 3 x 4
= 3 x (3 + 1)
= 3 x 3 + 3 x 1
= 1 x 3 x 3 + 1 x 3 + 0 x 1
= $1 \times 3^2 + 1 \times 3^1 + 0 \times 3^0$
= $110_{[3]}$

So we get 12 = $110_{[3]}$.
Thus, 12 in decimal is the same as 110 on the base 3.

Suggestions or Solutions

To the Problem 1

19 → [4]

19 in decimal has 19 of 1s.

In decimal system, we organize a number of 1s into bunches of 10s.

We want to convert 19 to a number on the base 4. And we can do it reorganizing 19 of 1s into bunches of 4s.

19

= 16 + 3

= 4 x 4 + 3

= 1 x 4 x 4 + 0 x 4 + 3 x 1

= 1 x 4^2 + 0 x 4^1 + 3 x 4^0

= $103_{[4]}$

Therefore, 19 = $103_{[4]}$.

We don't usually use the subscript for a decimal number.

Suggestions or Solutions

To the Problem 2

403 → [2]

403

$= 2 \times 201 + 1 = 2 \times (200 + 1) + 1$

$= 2 \times 200 + 2 \times 1 + 1 = \mathbf{2 \times 2} \times 100 + 2 + 1$

$= \mathbf{2^2} \times 2 \times 50 + 2 + 1 = 2^3 \times 2 \times 25 + 2 + 1$

$= 2^4 \times 25 + 2 + 1 = 2^4 \times (24 + 1) + 2 + 1$

$= 2^4 \times 24 + 2^4 \times 1 + 2 + 1 = 2^4 \times 2 \times 12 + 2^4 + 2 + 1$

$= 2^5 \times 12 + 2^4 + 2 + 1 = 2^5 \times 2 \times 6 + 2^4 + 2 + 1$

$= 2^6 \times 6 + 2^4 + 2 + 1 = 2^6 \times 2 \times 3 + 2^4 + 2 + 1$

$= 2^7 \times 3 + 2^4 + 2 + 1 = 2^7 \times (2 + 1) + 2^4 + 2 + 1$

$= 2^7 \times 2 + 2^7 \times 1 + 2^4 + 2 + 1 = 2^8 + 2^7 + 2^4 + 2 + 1$

$= 2^8 + 2^7 + 0 \times 2^6 + 0 \times 2^5 + 2^4 + 0 \times 2^3 + 0 \times 2^2 + 2^1 + 2^0$

$= 110010011_{[2]}$

Suggestions or Solutions

To the Problem 3

4305 → [5]

4305

$= 5 \times 861$

$= 5 \times (5 \times 172 + 1)$

$= 5^2 \times 172 + 5$

$= 5^2 \times (5 \times 34 + 2) + 5$

$= 5^3 \times 34 + 2 \times 5^2 + 5$

$= 5^3 \times (5 \times 6 + 4) + 2 \times 5^2 + 5$

$= 5^4 \times 6 + 4 \times 5^3 + 2 \times 5^2 + 5$

$= 5^4 \times (5 + 1) + 4 \times 5^3 + 2 \times 5^2 + 5$

$= 5^5 + 1 \times 5^4 + 4 \times 5^3 + 2 \times 5^2 + 5$

$= 1 \times 5^5 + 1 \times 5^4 + 4 \times 5^3 + 2 \times 5^2 + 1 \times 5^1 + 0 \times 5^0$

$= 114210_{[5]}$

Suggestions or Solutions
To the Problem 4

87 → [7]

1234 on the base 10

$= 1 \times 10^3 + 2 \times 10^2 + 3 \times 10^1 + 4 \times 10^0$

$= 1 \times 10^3 + 2 \times 10^2 + 3 \times 10 + 4 \times 1$

87

$= 7 \times 12 + 3$

$= 7 \times (7 + 5) + 3$

$= 7^2 + 5 \times 7 + 3$

$= 1 \times 7^2 + 5 \times 7^1 + 3 \times 7^0$

$= 153_{[7]}$

Suggestions or Solutions
To the Problem 5

101101 → [2]

101101

$= 2 \times 50550 + 1$

$= 2 \times (2 \times 25275) + 1$

$= 2^2 \times 25275 + 1$

$= 2^2 \times (2 \times 12637 + 1) + 1$

$= 2^3 \times 12637 + 2^2 + 1$

$= 2^3 \times (2 \times 6318 + 1) + 2^2 + 1$

$= 2^4 \times 6318 + 2^3 + 2^2 + 1$

$= 2^4 \times (2 \times 3159) + 2^3 + 2^2 + 1$

$= 2^5 \times 3159 + 2^3 + 2^2 + 1$

$= 2^5 \times (2 \times 1579 + 1) + 2^3 + 2^2 + 1$

$= 2^6 \times 1579 + 2^5 + 2^3 + 2^2 + 1$

$= 2^6 \times (2 \times 789 + 1) + 2^5 + 2^3 + 2^2 + 1$

$= 2^7 \times 789 + 2^6 + 2^5 + 2^3 + 2^2 + 1$

$= 2^7 \times (2 \times 394 + 1) + 2^6 + 2^5 + 2^3 + 2^2 + 1$

$= 2^8 \times 394 + 2^7 + 2^6 + 2^5 + 2^3 + 2^2 + 1$

$= 2^8 \times (2 \times 197) + 2^7 + 2^6 + 2^5 + 2^3 + 2^2 + 1$

$= 2^9 \times 197 + 2^7 + 2^6 + 2^5 + 2^3 + 2^2 + 1$

$= 2^9 \times (2 \times 98 + 1) + 2^7 + 2^6 + 2^5 + 2^3 + 2^2 + 1$

$= 2^{10} \times 98 + 2^9 + 2^7 + 2^6 + 2^5 + 2^3 + 2^2 + 1$

$= 2^{10} \times (2 \times 49) + 2^9 + 2^7 + 2^6 + 2^5 + 2^3 + 2^2 + 1$

$= 2^{11} \times (2 \times 24 + 1) + 2^9 + 2^7 + 2^6 + 2^5 + 2^3 + 2^2 + 1$

$= 2^{12} \times 24 + 2^{11} + 2^9 + 2^7 + 2^6 + 2^5 + 2^3 + 2^2 + 1$

$= 2^{12} \times (2 \times 12) + 2^{11} + 2^9 + 2^7 + 2^6 + 2^5 + 2^3 + 2^2 + 1$

$= 2^{13} \times 12 + 2^{11} + 2^9 + 2^7 + 2^6 + 2^5 + 2^3 + 2^2 + 1$

$= 2^{13} \times (2 \times 6) + 2^{11} + 2^9 + 2^7 + 2^6 + 2^5 + 2^3 + 2^2 + 1$

$= 2^{14} \times 6 + 2^{11} + 2^9 + 2^7 + 2^6 + 2^5 + 2^3 + 2^2 + 1$

$= 2^{15} \times 3 + 2^{11} + 2^9 + 2^7 + 2^6 + 2^5 + 2^3 + 2^2 + 1$

$= 2^{16} + 2^{15} + 2^{11} + 2^9 + 2^7 + 2^6 + 2^5 + 2^3 + 2^2 + 2^0$

$= 11000101011101101_{[2]}$

Suggestions or Solutions

To the Problem 6

702572 → [8]

702572

$= 8 \times 87821 + 4$

$= 8 \times (8 \times 10977 + 5) + 4$

$= 8^2 \times 10977 + 5 \times 8 + 4$

$= 8^2 \times (8 \times 1372 + 1) + 5 \times 8 + 4$

$= 8^3 \times 1372 + 8^2 + 5 \times 8 + 4$

$= 8^3 \times (8 \times 171 + 4) + 8^2 + 5 \times 8 + 4$

$= 8^4 \times (8 \times 171) + 4 \times 8^3 + 8^2 + 5 \times 8 + 4$

$= 8^5 \times (8 \times 21 + 3) + 4 \times 8^3 + 8^2 + 5 \times 8 + 4$

$= 8^6 \times 21 + 3 \times 8^5 + 4 \times 8^3 + 8^2 + 5 \times 8 + 4$

$= 8^6 \times (8 \times 2 + 5) + 3 \times 8^5 + 4 \times 8^3 + 8^2 + 5 \times 8 + 4$

$= 8^9 \times 2 + 5 \times 8^6 + 3 \times 8^5 + 4 \times 8^3 + 8^2 + 5 \times 8^1 + 4 \times 8^0$

$= 2005304154_{[8]}$

Suggestions or Solutions

To the Problem 7

11111→ [9]

11111

$= 9 \times 1234 + 5$

$= 9 \times (9 \times 137 + 1) + 5$

$= 9^2 \times 137 + 9 + 5$

$= 9^2 \times (9 \times 15 + 2) + 9 + 5$

$= 9^3 \times 15 + 2 \times 9^2 + 9 + 5$

$= 9^3 \times (9 + 6) + 2 \times 9^2 + 9 + 5$

$= 9^4 + 6 \times 9^3 + 2 \times 9^2 + 1 \times 9^1 + 5 \times 9^0$

$= 16215_{[9]}$

11111

$= 9^4 + 6 \times 9^3 + 2 \times 9^2 + 1 \times 9^1 + 5 \times 9^0$

$= 9(9^3 + 6 \times 9^2 + 2 \times 9 + 1) + 5$

$= 9(9(9^2 + 6 \times 9 + 2) + 1) + 5$

$= 9(9(9(9 + 6) + 2) + 1) + 5$

$= 9(9(9(9 \times 1 + 6) + 2) + 1) + 5$

Suppose we keep dividing 11111 by 9 and collect the remainder at a time. In other words, we begin the division with 9(9(9(9 x 1 + 6) + 2) + 1) + 5.

Then, the remainders are 5, 1, 2, 6, and 1.

So we can get the conversion if we arrange the remainders in the reverse order.

Therefore, we often use the method as follows:

```
9 | 11111
  ---------
9 | 1234  | 5
  --------
9 | 137  | 1
  -------
9 | 15  | 2
  ------
  | 1  | 6
```

In the procedure above, 1234, 137, 15, and 1 are the quotients and 5, 1, 2, and 6 are remainders.

Thus, 11111 in decimal is 16215 on the base 9.

We can see those quotients and the remainders in the processes as follows.

11111

$= 9 \times \mathbf{1234} + \mathbf{5}$

$= 9 \times (9 \times \mathbf{137} + \mathbf{1}) + 5$

$= 9^2 \times 137 + 9 + 5$

$= 9^2 \times (9 \times \mathbf{15} + \mathbf{2}) + 9 + 5$

$= 9^3 \times 15 + 2 \times 9^2 + 9 + 5$

$= 9^3 \times (9 + \mathbf{6}) + 2 \times 9^2 + 9 + 5$

$= 9^3 \times (9 \times \mathbf{1} + \mathbf{6}) + 2 \times 9^2 + 9 + 5$

$= 9^4 + 6 \times 9^3 + 2 \times 9^2 + 1 \times 9^1 + 5 \times 9^0$

$= 16215_{[9]}$

Suggestions or Solutions

To the Problem 8

10101→ [11]

10101

$= 11 \times 918 + 3$

$= 11 \times (11 \times 83 + 5) + 3$

$= 11^2 \times 83 + 5 \times 11 + 3$

$= 11^2 \times (11 \times 7 + 6) + 5 \times 11 + 3$

$= 11^3 \times 7 + 6 \times 11^2 + 5 \times 11^1 + 3 \times 11^0$

$= 7653_{[11]}$

Suggestions or Solutions

To the Problem 9

3462654 → [12]

3462654

$= 12 \times 288554 + 6$

$= 12 \times (12 \times 24046 + 2) + 6$

$= 12^2 \times 24046 + 2 \times 12 + 6$

$= 12^2 \times (12 \times 2003 + 10) + 2 \times 12 + 6$

$= 12^3 \times 2003 + 10 \times 12^2 + 2 \times 12 + 6$

$= 12^3 \times (12 \times 166 + 11) + 10 \times 12^2 + 2 \times 12 + 6$

$= 12^4 \times 166 + 11 \times 12^3 + 10 \times 12^2 + 2 \times 12 + 6$

$= 12^4 \times (12 \times 13 + 10) + 11 \times 12^3 + 10 \times 12^2 + 2 \times 12 + 6$

$= 12^5 \times 13 + 10 \times 12^4 + 11 \times 12^3 + 10 \times 12^2 + 2 \times 12 + 6$

$= 12^6 + 12^5 + 10 \times 12^4 + 11 \times 12^3 + 10 \times 12^2 + 2 \times 12 + 6$

$= 12^6 + 12^5 + A \times 12^4 + B \times 12^3 + A \times 12^2 + 2 \times 12^1 + 6 \times 12^0$

$= 11ABA26_{[12]}$

Suggestions or Solutions

To the Problem A

752015 → [14]

The alphabets are 0, 1, 2 . . . 9, A, B, C, and D. So to begin with we assign 10, 11, 12, and 13 to A, B, C, and D respectively in the same order.

752015

$= 14 \times 53715 + 5$

$= 14 \times (14 \times 3836 + 11) + 5$

$= 14^2 \times 3836 + 11 \times 14 + 5$

$= 14^2 \times (14 \times 274) + 11 \times 14 + 5$

$= 14^3 \times 274 + 11 \times 14 + 5$

$= 14^3 \times (14 \times 19 + 8) + 11 \times 14 + 5$

$= 14^4 \times 19 + 8 \times 14^3 + 11 \times 14 + 5$

$= 14^5 + 5 \times 14^4 + 8 \times 14^3 + 11 \times 14^1 + 5 \times 14^0$

$= 1580B5_{[14]}$

Suggestions or Solutions
To the Problem B

20022403 → [16]

20022403

$= 16 \times 1251400 + 3$

$= 16 \times (16 \times 78212 + 8) + 3$

$= 16^2 \times 78212 + 8 \times 16 + 3$

$= 16^2 \times (16 \times 4888 + 4) + 8 \times 16 + 3$

$= 16^3 \times 4888 + 4 \times 16^2 + 8 \times 16 + 3$

$= 16^3 \times (16 \times 305 + 8) + 4 \times 16^2 + 8 \times 16 + 3$

$= 16^4 \times 305 + 8 \times 16^3 + 4 \times 16^2 + 8 \times 16 + 3$

$= 16^4 \times (16 \times 19 + 1) + 8 \times 16^3 + 4 \times 16^2 + 8 \times 16 + 3$

$= 16^5 \times 19 + 16^4 + 8 \times 16^3 + 4 \times 16^2 + 8 \times 16 + 3$

$= 16^6 + 3 \times 16^5 + 16^4 + 8 \times 16^3 + 4 \times 16^2 + 8 \times 16^1 + 3 \times 16^0$

$= 1318483_{[16]}$

Suggestions or Solutions

To the Problem C

82031724 → [17]

82031724

$= 17 \text{ x } 4825395 + 9$

$= 17 \text{ x } (17 \text{ x } 283846 + 13) + 9$

$= 17^2 \text{ x } 283846 + 13 \text{ x } 17 + 9$

$= 17^2 \text{ x } (17 \text{ x } 16696 + 14) + 13 \text{ x } 17 + 9$

$= 17^3 \text{ x } 16696 + 14 \text{ x } 17^2 + 13 \text{ x } 17 + 9$

$= 17^3 \text{ x } (17 \text{ x } 982 + 2) + 14 \text{ x } 17^2 + 13 \text{ x } 17 + 9$

$= 17^4 \text{ x } 982 + 2 \text{ x } 17^3 + 14 \text{ x } 17^2 + 13 \text{ x } 17 + 9$

$= 17^4 \text{ x } (17 \text{ x } 57 + 13) + 2 \text{ x } 17^3 + 14 \text{ x } 17^2 + 13 \text{ x } 17 + 9$

$= 17^5 \text{ x } 57 + 13 \text{ x } 17^4 + 2 \text{ x } 17^3 + 14 \text{ x } 17^2 + 13 \text{ x } 17 + 9$

$= 17^5 \text{ x } (17\text{x}3 + 6) + 13\text{x}17^4 + 2\text{x}17^3 + 14\text{x}17^2 + 13\text{x}17 + 9$

$= 17^6 \text{ x } 3 + 6 \text{ x } 17^5 + 13 \text{ x } 17^4 + 2 \text{ x } 17^3 + 14 \text{ x } 17^2$

$+ 13 \text{ x } 17^1 + 9 \text{ x } 17^0$

$= 36D2ED9_{[17]}$

Suggestions or Solutions

To the Problem D

3054802 → [24]

3054802

$= 24 \times 127283 + 10$

$= 24 \times (24 \times 5303 + 11) + 10$

$= 24^2 \times 5303 + 11 \times 24 + 10$

$= 24^2 \times (24 \times 220 + 23) + 11 \times 24 + 10$

$= 24^3 \times 220 + 23 \times 24^2 + 11 \times 24 + 10$

$= 24^3 \times (24 \times 9 + 4) + 23 \times 24^2 + 11 \times 24 + 10$

$= 24^4 \times 9 + 4 \times 24^3 + 23 \times 24^2 + 11 \times 24^1 + 10 \times 24^0$

= 94NBA