Examples 5 in The Radix

Click this to start.

Examples 5 in The Radix

Convert the decimal numbers into the numbers on the bases indicated inside the square brackets.
Examples:

3 → [3] $3 = 10_{[3]}$

0.375 → [4] $0.375 = 0.12_{[4]}$

7.68 → [5] $7.68 = 12.32_{[5]}$

231/36 → [6] $231/36 = 10.23_{[6]}$

0. 243.7136 → [5]

1. 35.6875 → [2]

2. 702.244140625 → [8]

3. 100.69921875 → [5]

4. Convert 12.323 to the number on base 3, but find all the numbers in the digits above the dot and in the first seven digits below the dot.

5. Convert 10.101 to the number on base 4, but find all the numbers in the digits above the dot and in the first seven digits below the dot.

6. Convert 75.572 to the number on base 8, but find all the numbers in the digits above the dot and in the first seven digits below the dot.

7. Convert 111.11 to the number on base 9, but find all the numbers in the digits above the dot and in the first seven digits below the dot.

8. Convert 10.101 to the number on base 11, but find all the numbers in the digits above the dot and in the first seven digits below the dot.

9. Convert 3462.654 to the number on base 12, but find all the numbers in the digits above the dot and in the first five digits below the dot.

A. Convert 752.015 to the number on base 14, but find all the numbers in the digits above the dot and in the first 5 digits below the dot.

B. Convert 2002.2403 to the number on base 16, but find all the numbers in the digits above the dot and in the first seven digits below the dot.

C. Convert 8203.1724 to the number on base 17, but find all the numbers in the digits above the dot and in the first 4 digits below the dot.

D. Convert 305.4802 to the number on base 24, but find all the numbers in the digits above the dot and in the first 3 digits below the dot.

Suggestions or Solutions
To the Problem 0

243.7136 → [5]

Since 243.7136 is in decimal, that is, on the base 10, we can break it apart the way as follows:

243.7136

= 2 x 100 + 4 x 10 + 3 x 1

+ 7 x 1/10 + 1 x 1/100 + 3 x 1/1000 + 6/10000

= 2 x 10x10 + 4 x 10 + 3 x 1

+ 7 x 1/10 + 1/(10x10) + 3/(10x10x10) + 6/(10x10x10x10)

= 2 x 10^2 + 4 x 10 + 3 x 1

+ 7 x 1/10 + $1/10^2$ + 3 x $1/10^3$ + 6 x $1/10^4$

= 243.7136

A number on any base has the same structure.
Also, we can break 243.7236 into two parts as follows.
243.7236 = 243 + 0.7136.

So we can convert each of the two parts at a time.
We can begin with the digits above (or to the left of) the dot (or the point).

243
$= 5 \times 48 + 3$
$= 5 \times (5 \times 9 + 3) + 3$
$= 5^2 \times 9 + 3 \times 5 + 3$
$= 5^2 \times (5 + 4) + 3 \times 5 + 3$
$= 1 \times 5^3 + 4 \times 5^2 + 3 \times 5 + 3 \times 1$
$= 1433_{[5]}$.

Then, we want to convert the digits below (or to the right of) the dot. The digits are 7, 2, 3, and 6 in decimal, and we want to convert them to be on the base 5.
We know
$0.7236 = 7 \times 1/10 + 2 \times 1/10^2 + 3 \times 1/10^3 + 6 \times 1/10^4$.
So by the same token, we can set the equivalent number on the base 5 the way as follows.
$a \times 1/5 + b \times 1/5^2 + c \times 1/5^3 + \ d \times 1/5^4 + e \times 1/5^5 + \ldots$ where each of a, b, c, etc. is one of 0, 1, 2, 3, and 4, since the base is 5.

So we have

$a \times 1/5 + b \times 1/5^2 + c \times 1/5^3 + d \times 1/5^4 + e \times 1/5^5 + ...$

$= 0.abcde\ldots$ on base 5.

So we can set $0.7136_{[10]} = 0.abcde\ldots_{[5]}$.

Thus, we can set 0.7136

$= a \times 1/5 + b \times 1/5^2 + c \times 1/5^3 + d \times 1/5^4 + e \times 1/5^5 + ...$

So now, we need to find the values of a, b, c, d, e, etc.

Multiplying by 5 both sides of '=', we can get a.

First, multiplying the left hand side of '=' by 5, we get

$5 \times 0.7136 = 3.568$.

Next, multiplying the right hand side by 5, we get

$5 \times (a \times 1/5 + b \times 1/5^2 + c \times 1/5^3 + d \times 1/5^4 + e \times 1/5^5 + \ldots)$

$= a + b \times 1/5 + c \times 1/5^2 + d \times 1/5^3 + e \times 1/5^4 + f \times 1/5^5 + ...$

Then, we get

3.568

$= a + b \times 1/5 + c \times 1/5^2 + d \times 1/5^3 + e \times 1/5^4 + f \times 1/5^5 + ...$

So we've found the value of 'a', and a = 3.

Let's take 3 away from 3.568, and remove 'a' from

$a + b \times 1/5 + c \times 1/5^2 + d \times 1/5^3 + e \times 1/5^4 + f \times 1/5^5 + ...$

Then, setting

$0.568 = b \times 1/5 + c \times 1/5^2 + d \times 1/5^3 + e \times 1/5^4 + f \times 1/5^5 + ...$

and doing the same operation again, we can get the value of b.

So setting $5 \times 0.568 = 2.84$, and doing the operations, we get $2.84 = b + c \times 1/5 + d \times 1/5^2 + e \times 1/5^3 + ...$

Thus, we get b = 2.

So we can keep doing the same until we get all we need.

$5 \times 0.84 = 4.2$

$= c + d \times 1/5 + e \times 1/5^2 + f \times 1/5^3 + ...$ and c = 4.

$5 \times 0.2 = 1$

$= d + e \times 1/5 + f \times 1/5^2 + g \times 1/5^3 + ...$ and d = 1.

Now, it's done. What about e, f, g, etc.?

They are 0s. Thus, we get

$0.7136 = 3 \times 1/5 + 2 \times 1/5^2 + 4 \times 1/5^3 + 1/5^3 = 0.3241_{[5]}$.

Then, putting together the two parts, the integer and the fraction, we get $243.7136 = 1433.3241_{[5]}$.

Let's now see how the conversion can proceed the way above.
The conversion for each part goes the opposite way: multiplications vs. divisions.

Let's analyze the conversions on the two parts. One is above the dot, and the other is below the dot.

The part above (or to the left of) the dot is 243.

243
$= 5 \times 48 + 3$
$= 5 \times (5 \times 9 + 3) + 3$
$= 5^2 \times 9 + 3 \times 5 + 3$
$= 5^2 \times (5 + 4) + 3 \times 5 + 3$
$= 1 \times 5^3 + 4 \times 5^2 + 3 \times 5 + 3 \times 1$
$= 1433_{[5]}$.

And the part below (or to the right of) the dot is 0.7136.

0.7136

= (1/5)(5 x 0.7136)

= (1/5)(5 x (0.6 + 0.1136))

= (1/5)(3 + 5 x 0.1136)

= (1/5)(3 + 0.568)

= 3 x 1/5 + (1/5) x 0.568

= 3 x 1/5 + (1/5) x (1/5) x 5 x 0.568

= 3 x 1/5 + $(1/5^2)$ x 5 x (0.4 + 0.168)

= 3 x 1/5 + $(1/5^2)$ x (2 + 0.84)

= 3 x 1/5 + 2 x $(1/5^2)$ + $(1/5^2)$ x 0.84

= 3 x 1/5 + 2 x $(1/5^2)$ + $(1/5^2)$ x (1/5) x 5 x 0.84

= 3 x 1/5 + 2 x $(1/5^2)$ + $(1/5^3)$ x 5 x (0.8 + 0.04)

= 3 x 1/5 + 2 x $(1/5^2)$ + $(1/5^3)$ x (4 + 0.2)

= 3 x 1/5 + 2 x $(1/5^2)$ + 4 x $(1/5^3)$ + $(1/5^3)$ x 0.2

= 3 x 1/5 + 2 x $(1/5^2)$ + 4 x $(1/5^3)$ + $(1/5^3)$ x (1/5) x 5 x 0.2

= 3 x 1/5 + 2 x $(1/5^2)$ + 4 x $(1/5^3)$ + $(1/5^4)$ x 5 x 0.2

= 3 x 1/5 + 2 x $(1/5^2)$ + 4 x $(1/5^3)$ + $(1/5^4)$ x 1

= 3 x 1/5 + 2 x $(1/5^2)$ + 4 x $(1/5^3)$ + 1 x $(1/5^4)$

= $0.3241_{[5]}$.

So we can see that:

0.7136

$= 7 \times 1/10 + 1 \times 1/10^2 + 3 \times 1/10^3 + 6 \times 1/10^4$

$= 3 \times 1/5 + 2 \times (1/5^2) + 4 \times (1/5^3) + 1 \times (1/5^4)$

$= 0.3241_{[5]}$.

What we have done is that we have reorganized the amount less that 1 in decimal in terms of bunches of (1/5)s, $(1/5^2)$s, etc. We know that we put a decimal number in bunches of (1/10)s, $(1/10^2)$s, etc. when we show the digits below the dot.

Suggestions or Solutions
To the Problem 1

35.6875 → [2]

35.6875 = 35 + 0.6875

35 = 2 x 17 + 1 = 2 x (2 x 8 + 1) + 1
= 2 x (2 x 2 x 2 x 2 + 1) + 1
= 1 x 2^5 + 1 x 2 + 1 x 1
= $100011_{[2]}$.

We can set 0.6875 = a x 1/2 + b x $1/2^2$ + c x $1/2^3$ + ... where a, b, c, etc. are integers.
So now, we begin extracting the values of the a, b, c, etc.

2 x 0.6875 = 1.375
= 2 x (a x 1/2 + b x $1/2^2$ + c x $1/2^3$ +...)
= a + b x 1/2 + c x $1/2^2$ + d x $1/2^3$ +...
Thus, we can see that a = 1.

$2 \times 0.375 = 0.75$

$= b + c \times 1/2 + d \times 1/2^2 + e \times 1/2^3 + ...$

And we can see that $b = 0$.

$2 \times 0.75 = 1.5$

$= c + d \times 1/2 + e \times 1/2^2 + f \times 1/2^3 + ...$

And we can see that $c = 1$.

$2 \times 0.5 = 1$

$= d + e \times 1/2 + f \times 1/2^2 + ...$

And we can see that $d = 1$. Of course, e, f, g, etc. are all 0s.

Then, $35.6875 = 100011_{[2]}$

$+ (1 \times 1/2 + 0 \times 1/2^2 + 1 \times 1/2^3 + 1 \times 1/2^4)_{[10]}$

$= 100011.1011_{[2]}$.

So 35.6875 on the base 10 is the same as 100011.1011 on the base 2. Therefore, $19 = 103_{[4]}$.

Suggestions or Solutions

To the Problem 2

702.244140625 → [8]

702.422140625 = 702 + 0.422140625

702 = 8 x 87 + 6 = 8 x (8 x 10 + 7) + 6

= 8 x 8 x (8 + 2) + 8 x 7 + 6

= 1 x 8^3 + 2 x 8^2 + 7 x 8^1 + 6 x 8^0

= $1276_{[8]}$.

Setting 0.244140625

= a x 1/8 + b x $1/8^2$ + c x $1/8^3$ + ... where a, b, c, ... are integers, we get

8 x 0.244140625 = 1.9531258 = a + b x 1/8 + c x $1/8^2$ + ..., so we get a = 1. Next, we get

8 x 0. 9531258 = 7.625 = b + c x 1/8 + d x $1/8^2$ + ..., so we get b = 7. Next, we get

8 x 0.625 = 5 = c + d x 1/8 + e x $1/8^2$ + ..., so we get c = 5. And of course, d, e, etc. are 0s.

Thus, we get $702.244140625 = 1276.175_{[8]}$.

Suggestions or Solutions
To the Problem 3

100.69921875 → [4]

100 = 4 x 25 = 4 x (4 x 6 + 1) = 4 x 4 x (4 + 2) + 1 x 4 + 0 x 1
= 1 x 4x4x4 + 2 x 4x4 + 1 x 4 + 0 x 1 = $1210_{[4]}$.

And next, setting 0.69921875
= a x 1/4 + b x $1/4^2$ + c x $1/4^3$ + ..., where a, b, c, ... are integers, we can start extracting the values of the a, b, c, etc.

4 x 0.69921875 = 2.796875
= a + b x 1/4 + c x $1/4^2$ + d x $1/4^3$ + ... and a = 2.

4 x 0.796875 = 3.1875 = b + c x 1/4 + d x $1/4^2$ + ... and b = 3.
4 x 0.1875 = 0.75 = c + d x 1/4 + e x $1/4^2$ + ... and c = 0.

4 x 0.75 = 3 = d + e x 1/4 + f x $1/4^2$ + ...
And we get d = 3, and e, f, etc. are all 0s.
Thus, 100.69921875 = $1210.2303_{[4]}$.

Suggestions or Solutions
To the Problem 4

4. Convert 12.323 to the number in base 3, but find all the numbers in the digits above the dot and in the first seven digits below the dot.

To begin with,

$12 = 3 \times 4 = 3 \times 3 + 3 = 1 \times 3x3 + 1 \times 3 + 0 \times 1$

$= 1 \times 3^2 + 1 \times 3 + 0 \times 1 = 110_{[3]}$

Next, we can set $0.323 = a_1 \times 1/3 + a_2 \times 1/3^2 + a_3 \times 1/3^3 + ...$, where a_1, a_2, a_3, etc. are integers. Then, we get these:

$3 \times 0.323 = 0.969 = 3(a_1 \times 1/3 + a_2 \times 1/3^2 + a_3 \times 1/3^3 + ...)$

$= a_1 + a_2 \times 1/3 + a_3 \times 1/3^2 + a_4 \times 1/3^3 + ...$ and $a_1 = 0$.

$3 \times 0.969 = 2.907 = a_2 + a_3 \times 1/3 + a_4 \times 1/3^2 + ...$ and $a_2 = 2$.

$3 \times 0.907 = 2.721 = a_3 + a_4 \times 1/3 + a_5 \times 1/3^2 + ...$ and $a_3 = 2$.

$3 \times 0.721 = 2.163 = a_4 + a_5 \times 1/3 + a_6 \times 1/3^2 + \ldots$ and $a_4 = 2$.

$3 \times 0.163 = 0.489 = a_5 + a_6 \times 1/3 + a_7 \times 1/3^2 + \ldots$ and $a_5 = 0$.

$3 \times 0.489 = 1.467 = a_6 + a_7 \times 1/3 + a_8 \times 1/3^2 + \ldots$ and $a_6 = 1$.

$3 \times 0.467 = 1.401 = a_7 + a_8 \times 1/3 + a_9 \times 1/3^2 + \ldots$ and $a_7 = 1$.

Therefore, $12.323 = 110.0222011\ldots$ [3]

Suggestions or Solutions
To the Problem 5

5. Convert 10.101 to the number in base 4, but find all the numbers in the digits above the dot and in the first seven digits below the dot.

To begin with,
$10 = 2 \times 4 + 2 \times 1 = 22_{[4]}$

Next, we can set $0.101 = a \times 1/4 + b \times 1/4^2 + c \times 1/4^3 + ...$, where a, b, c, etc. are all integers. Then, we get these:

$4 \times 0.101 = 0.404 = 4(a \times 1/4 + b \times 1/4^2 + ...)$
$= a + b \times 1/4 + c \times 1/4^2 + ...$ and $a = 0$.

$4 \times 0.404 = 1.616 = b + c \times 1/4 + d \times 1/4^2 + ...$ and $b = 1$.

$4 \times 0.616 = 2.464 = c + d \times 1/4 + e \times 1/4^2 + ...$ and $c = 2$.

$4 \times 0.464 = 2.163 = d + e \times 1/3 + f \times 1/3^2 + ...$ and $d = 2$.

$4 \times 0.163 = 0.652 = e + f \times 1/3 + f \times 1/3^2 + \ldots$ and $e = 0$.

$4 \times 0.652 = 2.608 = f + g \times 1/3 + h \times 1/3^2 + \ldots$ and $f = 2$.

$4 \times 0.608 = 2.432 = g + h \times 1/3 + i \times 1/3^2 + \ldots$ and $g = 2$.

Therefore, 10.101 = 22. 0122022... [4].

Suggestions or Solutions
To the Problem 6

6. Convert 75.572 to the number in base 8, but find all the numbers in the digits above the dot and in the first seven digits below the dot.

To begin with,

$75 = 8 \times 9 + 3 = 8 \times (8 + 1) + 3 = 8 \times 8 + 8 + 3$

$= 113_{[8]}$.

Next, setting $0.572 = a \times 1/8 + b \times 1/8^2 + c \times 1/8^3 + ...$, where a, b, c, ... are integers, we can get these:

$8 \times 0.572 = 4.576$ and $a = 4$.

$8 \times 0.576 = 4.608$ and $b = 4$.

$8 \times 0.608 = 4.864$ and $c = 4$.

$8 \times 0.864 = 6.912$ and $d = 6$.

$8 \times 0.912 = 7.296$ and $e = 7$.

$8 \times 0.296 = 2.368$ and $f = 2$.

$8 \times 0.368 = 2.944$ and $g = 2$.

Therefore, $75.572 = 113.4446722..._{[8]}$.

Suggestions or Solutions

To the Problem 7

7. Convert 111.11 to the number in base 9, but find all the numbers in the digits above the dot and in the first seven digits below the dot.

$111 = 9 \times 12 + 3 = 9 \times (9 + 2) + 3 = 123_{[9]}$.

$9 \times 0.11 = 0.99 \rightarrow 0$

$9 \times 0.99 = 8.91 \rightarrow 8$

$9 \times 0.91 = 8.19 \rightarrow 8$

$9 \times 0.19 = 1.71 \rightarrow 1$

$9 \times 0.71 = 6.39 \rightarrow 6$

$9 \times 0.39 = 3.51 \rightarrow 3$

$9 \times 0.51 = 4.59 \rightarrow 4$

. . .

Therefore, $111.11 = 123.0881634..._{[9]}$.

Suggestions or Solutions
To the Problem 8

8. Convert 10.101 to the number in base 11, but find all the numbers in the digits above the dot and in the first seven digits below the dot.

$10 = A_{[11]}$.

11 x 0.101 = 1.111, 1
11 x 0.111 = 1.221, 1
11 x 0.221 = 2.431, 2
11 x 0.431 = 4.741, 4
11 x 0.741 = 8.151, 8
11 x 0.151 = 1.661, 1
11 x 0.661 = 7.271, 7
...

Therefore, $10.101 = A.1124817..._{[11]}$.

Suggestions or Solutions

To the Problem 9

9. Convert 3462.654 to the number in base 12, but find all the numbers in the digits above the dot and in the first five digits below the dot.

3462 = 12 x 288 + 6 = 12 x 12 x 24 + 6

= 2 x 12 x 12 x 12 + 6 = $2006_{[12]}$.

12 x 0.654 = 7.848, 7

12 x 0.848 = 10.176, A

12 x 0.176 = 2.112, 2

12 x 0.112 = 1.344, 1

12 x 0.344 = 4.128, 4

. . .

Therefore, 3462.654 = $2006.7A214..._{[12]}$.

Suggestions or Solutions
To the Problem A

A. Convert 752.015 to the number in base 14, but find all the numbers in the digits above the dot and in the first 5 digits below the dot.

$752 = 14 \times 53 + 10 + 14 \times (14 \times 3 + 5) + 10 = 35A_{[14]}$.

14 x 0.015 = 0.21, 0
14 x 0.21 = 2.94, 2
14 x 0.94 = 13.16, D
14 x 0.16 = 2.24, 2
14 x 0.24 = 3.36, 3
. . .

Therefore, $752.015 = 35A.02D23..._{[14]}$.

Suggestions or Solutions
To the Problem B

B. Convert 2002.2403 to the number in base 16, but find all the numbers in the digits above the dot and in the first seven digits below the dot.

$2002 = 16 \times 125 + 2 = 16 \times (16 \times 7 + 13) + 2 = 7D2_{[16]}$.

0.2403 x 16 = 3.8448, 3
0.8448 x 16 = 13.5168, D
0.5168 x 16 = 8.2688, 8
0.2688 x 16 = 4.3008, 4
0.3008 x 16 = 4.8128, 4
0.8128 x 16 = 13.0048, D
0.0048 x 16 = 0.0768, 0
0.0768 x 16 = 1.2288, 1
. . .

$2002.2403 = 7D2.3D844D0..._{[16]}$.

Suggestions or Solutions
To the Problem C

C. Convert 8203.1724 to the number in base 17, but find all the numbers in the digits above the dot and in the first 4 digits below the dot.

8203 = 17 x 482 + 9 = 17(17 x 28 + 6) + 9
= 17(17(17 + 9) + 6) + 9 = $1969_{[17]}$

0.1724 x 17 = 2.9308, 2
0.9308 x 17 = 15.8236, F
0.8236 x 17 = 14.0012, E
0.0012 x 17 = 0.0204, 0
. . .

Therefore, 8203.1724 = 1969.2FE0...$_{[17]}$.

Suggestions or Solutions
To the Problem D

D. Convert 305.4802 to the number in base 24, but find all the numbers in the digits above the dot and in the first 3 digits below the dot.

$305 = 24 \times 12 + 17 = CH_{[24]}$

0.4802 x 24 = 11.5248, B
0.5248 x 24 = 12.5952, C
0.5952 x 24 = 14.2848, E
. . .

Therefore, $305.4802 = CH.BCE..._{[24]}$.