

Examples 6 in The Radix

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Examples 6 in The Radix

Convert the decimal numbers into the numbers on the bases indicated inside the square brackets.

Examples:

Problem: $11_{[2]} \rightarrow [3]$ Solution: $11_{[2]} = 10_{[3]}$

Problem: $20_{[3]} \rightarrow [4]$ Solution: $20_{[3]} = 12_{[4]}$

Problem: $11_{[6]} \rightarrow [5]$ Solution: $11_{[6]} = 12_{[5]}$

Problem: $12_{[35]} \rightarrow [6]$ Solution: $12_{[35]} = 101_{[6]}$

0. $12_{[4]} \rightarrow [3]$ 1. $19_{[11]} \rightarrow [4]$ 2. $403_{[5]} \rightarrow [2]$

3. $4B3_{[12]} \rightarrow [7]$ 4. $BA7_{[15]} \rightarrow [5]$ 5. $101101_{[4]} \rightarrow [2]$

6. $112122_{[3]} \rightarrow [9]$ 7. $32331_{[6]} \rightarrow [9]$

8. $2361_{[9]} \rightarrow [3]$ 9. $220201_{[3]} \rightarrow [12]$

10. $AB9B_{[12]} \rightarrow [24]$ 11. $23203_{[4]} \rightarrow [16]$

12. $8AB34_{[16]} \rightarrow [4]$ 13. $9F9C2_{[16]} \rightarrow [24]$

Suggestions or Solutions To the Problem 0

0. $12_{[4]} \rightarrow [3]$

We can convert the number to decimal equivalent first, and then, convert the decimal to the number on the base.

Sometimes, we can convert the number directly to the equivalent number on the base without the conversion to the decimal.

$$\begin{aligned} &12_{[4]} \\ &= 1 \times 4 + 2 \times 1 \\ &= 4 + 2 \\ &= 6 \quad \{\text{Now, it is decimal.}\} \\ &= 2 \times 3 + 0 \times 1 \\ &= 20_{[3]} \end{aligned}$$

Suggestions or Solutions To the Problem 1

1. $19_{[11]} \rightarrow [4]$

$$19_{[11]}$$

$$= 1 \times 11 + 9 \times 1$$

$$= 11 + 9$$

$$= 20 \quad \{\text{It is decimal, now.}\}$$

$$= 4 \times 5$$

$$= 4 \times (4 + 1)$$

$$= 4 \times 4 + 4$$

$$= 1 \times 4 \times 4 + 1 \times 4 + 0 \times 1$$

$$= 110_{[4]}$$

Suggestions or Solutions To the Problem 2

2. $403_{[5]} \rightarrow [2]$

$$403_{[5]}$$

$$= 4 \times 5 \times 5 + 0 \times 5 + 3 \times 1$$

$$= 100 + 3$$

$$= 103 \quad \{\text{It is decimal, now.}\}$$

$$= 2 \times 51 + 1$$

$$= 2 \times (2 \times 25 + 1) + 1$$

$$= 2^2 \times 25 + 2 + 1$$

$$= 2^2 \times (2 \times 12 + 1) + 2 + 1$$

$$= 2^3 \times 12 + 2^2 + 2 + 1$$

$$= 2^3 \times (2 \times 6) + 2^2 + 2 + 1$$

$$= 2^4 \times 6 + 2^2 + 2 + 1$$

$$= 2^4 \times (2 \times 3) + 2^2 + 2 + 1$$

$$= 2^5 \times 3 + 2^2 + 2 + 1$$

$$= 2^5 \times (2 + 1) + 2^2 + 2 + 1$$

$$= 2^6 + 2^5 + 2^2 + 2 + 1$$

$$= 1100111_{[2]}$$

Suggestions or Solutions To the Problem 3

3. $4B3_{[12]} \rightarrow [2]$

$$4B3_{[12]}$$

$$= 4 \times 12^2 + B \times 12 + 3$$

$$= 4 \times 144 + 11 \times 12 + 3$$

$$= 576 + 132 + 3$$

$$= 711$$

$$= 7 \times 101 + 4$$

$$= 7 \times (7 \times 14 + 3) + 4$$

$$= 7^2 \times 14 + 7 \times 3 + 4$$

$$= 2 \times 7^3 + 3 \times 7 + 4$$

$$= 2034_{[7]}$$

Suggestions or Solutions To the Problem 4

4. $BA7_{[15]} \rightarrow [5]$

$$BA7_{[15]}$$

$$= B \times 15^2 + A \times 15 + 7$$

$$= 11 \times 15^2 + 10 \times 15 + 7$$

$$= 2475 + 150 + 7$$

$$= 2632$$

$$= 5 \times 526 + 2$$

$$= 5 \times (5 \times 105 + 1) + 2$$

$$= 5^2 \times (5 \times 21) + 5 + 2$$

$$= 5^3 \times (5 \times 4 + 1) + 5 + 2$$

$$= 4 \times 5^4 + 5^3 + 5 + 2$$

$$= 41012_{[5]}$$

Suggestions or Solutions To the Problem 5

5. $101101_{[4]} \rightarrow [2]$

We may want to try direct conversion for this problem.

$$\begin{aligned} &101101_{[4]} \\ &= 1 \times 4^5 + 1 \times 4^3 + 1 \times 4^2 + 1 \\ &= 1 \times (2^2)^5 + 2^6 + 2^4 + 1 \quad \{\text{We know } 4 = 2^2.\} \\ &= 2^{10} + 2^6 + 2^4 + 1 \\ &= 10001010001_{[2]} \end{aligned}$$

Suggestions or Solutions To the Problem 6

6. $112122_{[3]} \rightarrow [9]$

We may want to try direct conversion for this problem.

$$\begin{aligned} &112122_{[3]} \\ &= 3^5 + 3^4 + 2 \times 3^3 + 3^2 + 2 \times 3 + 2 \\ &= 3 \times 3^4 + 3^4 + 2 \times 3 \times 3^2 + 3^2 + 2 \times 3 + 2 \\ &= 4 \times 9^2 + 6 \times 9 + 9 + 6 + 2 \quad \{\text{We know } 3^2 = 9.\} \\ &= 4 \times 9^2 + 7 \times 9 + 8 \\ &= 478_{[9]} \end{aligned}$$

Suggestions or Solutions To the Problem 7

7. $32331_{[6]} \rightarrow [9]$

We may want to try direct conversion for this problem, tpo.

$$32331_{[6]}$$

$$= 3 \times 6^4 + 2 \times 6^3 + 3 \times 6^2 + 3 \times 6 + 1 \quad \{6 = 2 \times 3\}$$

$$= 3 \times (2 \times 3)^4 + 2 \times (2 \times 3)^3 + (2 \times 3)^2 + 2 \times 9 + 1 \quad \{3^2 = 9\}$$

$$= 3 \times 2^4 \times 9^2 + 2 \times 2^3 \times 3 \times 9 + 4 \times 9 + 2 \times 9 + 1$$

$$= 3 \times 16 \times 9^2 + (48 + 4 + 2) \times 9 + 1$$

$$= 48 \times 9^2 + 54 \times 9 + 1$$

$$= (5 \times 9 + 3) \times 9^2 + 6 \times 9 \times 9 + 1$$

$$= 5 \times 9^3 + (3 + 6) \times 9^2 + 1$$

$$= 6 \times 9^3 + 1$$

$$= 6001_{[9]}$$

Suggestions or Solutions
To the Problem 8

8. $2361_{[9]} \rightarrow [3]$

$$\begin{aligned} &2361_{[9]} \\ &= 2 \times 9^3 + 3 \times 9^2 + 6 \times 9 + 1 \\ &= 2 \times 3^6 + 3^5 + 2 \times 3^3 + 1 \\ &= 2102001_{[3]} \end{aligned}$$

Suggestions or Solutions To the Problem 9

9. $220201_{[3]} \rightarrow [12]$

$$220201_{[3]}$$

$$= 2 \times 3^5 + 2 \times 3^4 + 2 \times 3^2 + 1$$

$$= 2 \times 3 \times 3^4 + 2 \times 3^4 + 18 + 1$$

$$= (6 + 2) \times 3^4 + 12 + 7$$

$$= 8 \times 3 \times 27 + 12 + 7$$

$$= 24 \times (24 + 3) + 12 + 7$$

$$= 2 \times 12 \times (2 \times 12 + 3) + 12 + 7$$

$$= 4 \times 12^2 + 6 \times 12 + 12 + 7$$

$$= 4 \times 12^2 + 7 \times 12 + 7$$

$$= 477_{[12]}$$

Suggestions or Solutions To the Problem A

A. AB9B_[12] → [24]

$$AB9B_{[12]} = 10 \times 12^3 + 11 \times 12^2 + 9 \times 12 + 11$$

$$= 2 \times 5 \times 12 \times 12 \times 2 \times 6 + 11 \times 12 \times 2 \times 6 + 9 \times 12 + 11$$

$$= 5 \times 24 \times 6 \times 24 + 66 \times 24 + 9 \times 12 + 12 - 1$$

$$= 30 \times 24^2 + 66 \times 24 + 10 \times 12 - 1$$

$$= (24 + 6) \times 24^2 + (48 + 18) \times 24 + 5 \times 24 - 1$$

$$= 24^3 + 6 \times 24^2 + 2 \times 24^2 + 18 \times 24 + 4 \times 24 + 24 - 1$$

$$= 24^3 + 8 \times 24^2 + 22 \times 24 + 23$$

$$= 18MN_{[24]}$$

**Suggestions or Solutions
To the Problem B**

B. $23203_{[4]} \rightarrow [16]$

$$23203_{[4]}$$

$$= 2 \times 4^4 + 3 \times 4^3 + 2 \times 4^2 + 3$$

$$= 2 \times 16^2 + 12 \times 16 + 2 \times 16 + 3$$

$$= 2 \times 16^2 + 14 \times 16 + 3$$

$$= 2E3_{[16]}, \text{ since E has 14 of 1s.}$$

Suggestions or Solutions To the Problem C

C. $8AB34_{[16]} \rightarrow [4]$

$$8AB34_{[16]}$$

$$= 8 \times 16^4 + 10 \times 16^3 + 11 \times 16^2 + 3 \times 16 + 4$$

$$= 2 \times 4 \times 4^8 + (2 \times 4 + 2) \times 4^6 + (2 \times 4 + 3) \times 4^4 + 3 \times 4^2 + 4$$

$$= 2 \times 4^9 + 2 \times 4^7 + 2 \times 4^6 + 2 \times 4^5 + 3 \times 4^4 + 3 \times 4^2 + 4$$

$$= 2022230310_{[4]}$$

Suggestions or Solutions To the Problem D

D. 9F9C2_[16] → [24]

$$9F9C2_{[16]} = 9 \times 16^4 + 15 \times 16^3 + 9 \times 16^2 + 12 \times 16 + 2$$

$$= 3^2 \times 2^4 \times 8^4 + 3 \times 5 \times 2^3 \times 8^3 + (3 \times 3 \times 2 \times 8)^2 + 2$$

$$= 2^4 \times 8^2 \times 24^2 + 5 \times 2^3 \times 8^2 \times 24 + 36 \times 24^2 + 2$$

$$= 32^2 \times 24^2 + 5 \times 8^3 \times 24 + (24 + 12) \times 24^2 + 2$$

$$= 1024 \times 24^2 + 2560 \times 24 + 24^3 + 12 \times 24^2 + 2$$

$$= (42 \times 24 + 16) \times 24^2 + (106 \times 24 + 16) \times 24$$

$$+ 24^3 + 12 \times 24^2 + 2$$

$$= 42 \times 24^3 + 16 \times 24^2 + 106 \times 24^2 + 24^3 + 12 \times 24^2 + 2$$

$$= 43 \times 24^3 + 134 \times 24^2 + 2$$

$$= (24 + 19) \times 24^3 + (5 \times 24 + 14) \times 24^2 + 2$$

$$= 24^4 + 19 \times 24^3 + 5 \times 24^3 + 14 \times 24^2 + 2$$

$$= 2 \times 24^4 + 5 \times 24^3 + 14 \times 24^2 + 2$$

$$= 25E02_{[24]}$$