Examples 7 in The Radix

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Convert the decimal numbers into the numbers on the bases indicated inside the square brackets.

Examples:

Problem: $11_{[2]} \rightarrow [3]$ Solution: $11_{[2]} = 10_{[3]}$

Problem: $0.011_{[2]} \rightarrow [4]$ Solution: $0.011_{[2]} = 0.12_{[4]}$

0. $1.2_{[4]} \rightarrow [2]$

1. $1.9_{[11]} \rightarrow [4]$

2. $36.2163_{[9]} \rightarrow [3]$

3. $5.A93_{[12]} \rightarrow [3]$

4. $0.1_{[3]} \rightarrow [9]$

5. $0.101_{[2]} \rightarrow [4]$

6. $0.4872_{[9]} \rightarrow [3]$

7. 0.42315[6] $\rightarrow$ [12]

Suggestions or Solutions

To the Problem 0

0. $1.2_{[4]} \rightarrow [2]$

1.2

= 1 + 2/4 {We know 1.2 has 1 of 1s and 2 of (1/4)s.}

= 1 + 1/2

= 1 x 1 + 1 x 1/2

= $1.1_{[2]}$

Suggestions or Solutions

To the Problem 1

1. $1.9_{[11]} \rightarrow [4]$

$1.9_{[11]} = 1 + 9 \times 1/11 = 1 + 9/11$

$1 = 1_{[4]}$

We can set $9/11 = a/4 + b/4^2 + c/4^3 + \ldots$ where a, b, c, etc. are integers. Then, we can get these:

$(9/11) \times 4 = \mathbf{36/11} = 3 + 3/11 = a + b/4 + c/4^2 + d/4^3 + \ldots$ and a = 3.

$(3/11) \times 4 = 12/11 = 1 + 1/11 = b + c/4 + d/4^2 + \ldots$ and b = 1.

$(1/11) \times 4 = 4/11 = 0 + 4/11$, and c = 0.

$(4/11) \times 4 = 16/11 = 1 + 5/11$, and d = 1.

4 x 5/11 = 20/11 = 1 + 9/11, and e = 1.

4 x 9/11 = **36/11** = 3 + 3/11, and f = 3.

Now, we know 36/11 has shown up already. So the group of numbers (3, 1, 0, 1, and 1) is going to repeat itself indefinitely. So, we get

$1.9_{[11]}$
= 1 + 9/11 in decimal
= $1.31011310113101131011..._{[11]}$

This kind of repetition can happen as in the case of a rational number such as 4/3 = 1.33333...

Suggestions or Solutions

To the Problem 2

$36.2163_{[9]} \rightarrow [3]$

$36_9 = 27 + 6 = 33 = 3 \times 10 + 3 = 3 \times (9 + 1) + 3 = 3^3 + 2 \times 3$

$= 1000_3 + 20_3 = 1020_3$

$0.2163_{[9]}$

$= 2 \times 1/9 + 1/9^2 + 6/9^3 + 3/9^4$

$= 2/3^2 + 1/3^4 + 6/3^6 + 3/3^8$ {We know $1/9 = 1/3^2$.}

$= 2/3^2 + 1/3^4 + 2/3^5 + 1/3^7$

$= 0.0201201_{[3]}$

So we get $36.2163_{[9]} = 1020.0201201_{[3]}$.

Suggestions or Solutions

To the Problem 3

$5.A93_{[12]} \rightarrow [3]$

$5.A93_{[12]} = 5 + 10/12 + 9/12^2 + 3/12^3$

$5_{[12]} = 3 + 2 = 12_{[3]}$

$0.A93_{[12]}$

$= 10/12 + 9/12^2 + 3/12^3$

$= (10 \times 12^2 + 9 \times 12 + 3)/12^3$

$= (1440 + 108 + 3)/1728$

$= 1551/1728$

[Note that 1 + 5 + 5 + 1 = 12, and 1 + 7 + 2 + 8 = 18. So 1551 and 1728 are multiples of 3.]

$= (3 \times 517)/(3 \times 576) = 517 / 576$

Let’s now, extract the digits below the dot on the base 3.

3 x 517/576 = 1551/576 = 2 + 399/576 → 2

3 x 399/576 = 1597/576 = 2 + 445/576 → 2

3 x 445/576 = 1335/576 = 2 + 183/576 → 2

3 x 183/576 = 549/576 = 0 + 549/576 → 0

3 x 549/576 = 1647/576 = 2 + 496/576 → 2

3 x 496/576 = 1488/576 = 2 + 336/576 → 2

3 x 336/576 = 1008/576 = 1 + **432/576** → 1

3 x 432/576 = 1296/576 = 2 + 144/576 → 2

3 x 144/576 = 0 + **432/576** → 0

Now, we know 432/576 has shown up already. So the group of numbers (2 and 0) is going to repeat itself after 2220221 indefinitely.

So we get $0.A93_{[12]} = 0.222022120202020\ldots_{[3]}$.

Thus, we get

$5.A93_{[12]} = 5 + 517/576 = 12.2220221202020\ldots_{[3]}$.

Suggestions or Solutions

To the Problem 4

$0.1_{[3]} \rightarrow [9]$

$0.1_{[3]}$ {0.1 on the base 3 has 1 of 1/3.}

= 1 x 1/3

= 1/3

= 3/9 {1/3 = 3/9}

= 3 x 1/9 {3/9 has 3 of (1/9)s, and 0.3 on the base 9 has 3 of (1/9)s.}

= $0.3_{[9]}$.

Suggestions or Solutions

To the Problem 5

$0.101_{[2]} \rightarrow [4]$

$0.101_{[2]}$

$= 1/2 + 0/2^2 + 1/2^3$

$= 2/4 + 1/8$

$= 2/4 + 2/16$

$= 2/4 + 2/4^2$

$= 0.22_{[4]}$.

Suggestions or Solutions
To the Problem 6

$0.4872_{[9]} \rightarrow [3]$

$0.4872_{[9]} = 4/9 + 8/9^2 + 7/9^3 + 2/9^4$

$4/9 = (3 +1)/9 = 3/9 + 1/9 = 1/3 + 1/3^2$

$8/9^2 = (6 + 2)/9^2 = 6/9^2 + 2/9^2 = (2x3)/3^4 + 2/3^4 = 2/3^3 + 2/3^4$

$7/9^3 = (6 + 1)/9^3 = 6/9^3 + 1/9^3$

$= (2 \times 3)/3^6 + 1/3^6 = 2/3^5 + 1/3^6$

$2/9^4 = 2/3^8$

Therefore, we get

$0.4872_{[9]} = 4/9 + 8/9^2 + 7/9^3 + 2/9^4$

$= 1/3 + 1/3^2 + 2/3^3 + 2/3^4 + 2/3^5 + 1/3^6 + 2/3^8$

$= 0.11222102_{[3]}$

Suggestions or Solutions

To the Problem 7

$0.42315_{[6]} \rightarrow [12]$

$0.42315_{[6]} = 4/6 + 2/6^2 + 3/6^3 + 1/6^4 + 5/6^5$

$= 8/12 + (2 \times 2^2)/12^2 + (3 \times 2^3)/12^3 + 2^4/12^4 + (5 \times 2^5)/12^5$

$(2 \times 2^2)/12^2 = 8/12^2$

$(3 \times 2^3)/12^3 = 24/12^3 = 2/12^2$

$2^4/12^4 = 16/12^4 = (12 + 4)/12^4 = 1/12^3 + 4/12^4$

$(5 \times 2^5)/12^5 = 160/12^5 = (12 \times 13 + 4)/12^5$

$= 12(12 + 1)/12^5 + 4/12^5 = 1/12^3 + 1/12^4 + 4/12^5$

Therefore,

$0.42315_{[6]} = 4/6 + 2/6^2 + 3/6^3 + 1/6^4 + 5/6^5$

$= 8/12 + 8/12^2 + 2/12^2 + 1/12^3 + 4/12^4 + 1/12^3 + 1/12^4$

$+ 4/12^5$

$= 8/12 + 10/12^2 + 2/12^3 + 5/12^4 + 4/12^5$

$= 0.8A254_{[12]}$