Dear students,

Students need the best teacher, so you need examples, because examples are the best teacher. This is thus, a place of examples. All the examples here are fully worked, and explain **how** the basic but essential tools in math are made, together with **what** they are, **how** they work, and **how** to work with them. Such tools include numbers, formulas, identities, equations, laws, etc.

Doing math, we work with ideas and run ideas, because every thing in math is an idea. What idea though?

It's a concept. So we need to understand it if we want to use it. A number is an idea, for instance, and the same is true for a line or circle, too. And putting ideas together, we can build another, which becomes the base or an element of another, and each is connected. And that's the way your math grows.

So you get to build a circuit, and sometimes, need to fill the gap or repair the circuit so that you can get the sense of it. Your calculation thus, runs properly, and gets to the solution.

Getting the concept doing good examples, and gaining the fluency doing right examples, you can build the circuit strong.

This place is however, nothing but a bunch of examples until you get it powered. How then to get it powered, and make it run and work for you?

Read examples closely at your pace, and then, do each example yourself. It is quite important to note that you do it in **your** own writing. Just watching someone doing it, you just only feel that you can do it, too, but can hardly do it when you need it. You do it, you get it. And you've got to do it over with different examples. It's a cliché, of course, but is always true that knowing is one thing and doing is another.

I've been helping students get conceptual understanding and fluency so that they can grow, take care of, and run their own math. The area covers the basics in algebra and geometry, and includes equations for unknowns and those for curves, functions and their graphs, and other math tools, which are the basic elements in calculus, which's been the core of my interest from my early age in high school.

Of my students, some are quite poor in math, and thus, are afraid of or even hate math, some require special education because of exceptional intelligence, some are smart enough, some are naïve and diligent, and many are clever but lazy. All the students are badly after though, one thing in common.

A Strong Foundation in Algebra

It is of course, the prime objective of my work, and I'm always happy and eager to help them achieve it. I can only help though. Your math grows as much as you grow and take care of it yourself. So does mine. What math then do students most often do or use in high schools or colleges?

Algebra and geometry. What algebra then?

Elementary algebra, of course. Doing the algebra, you should be able to not only work with, but manipulate, that is, change, alter, and convert math expressions the way you want, so for instance, you can break apart and factor polynomials in many kinds. Also, you can handle properly ratios and rates, trigonometry, factors or divisors, constants and variables, powers and logarithms or exponents, equations and functions, along with their graphs, equalities and inequalities, identities and formulas, laws and rules, etc.

And if you want to handle them properly, you want to know their natures and how they mingle with each other.

So studying those stated above, you want to know **what** they are, **how** they work, and **how** to work with them or **what** to do with them. What then about the geometry?

Basically, the geometry is about shapes, and has much to do with positions and angles in algebra.   Why algebra?

You often need to compute lengths, areas, or volumes in many situations. The shapes begin with points, lines, triangles, and circles, and move on to rectangles, squares, parallelograms or rhombuses, trapezoids, tetragons, other polygons as pentagons, polyhedrons as hexahedrons, etc.

Doing the geometry, too, we need to do the algebra stated above such as trigonometry. So it is analytic geometry, often called coordinate geometry, too. And doing it, we can specify positions using coordinates. So in that geometry, basically, we work with graphs. Putting a math idea in a graph, we can not only effectively think about it, but actually see it, too, and therefore, can not just easily, but efficiently work with it, too. What idea then is it?

The idea begins with a point, a line, a parabola, a circle, an ellipse, and a hyperbola, called a conic section or basic curve, and then, moves on to other curves, planes, surfaces, volumes, and other objects in various dimensional spaces, together with vectors. And using an angle, we can specify and work with an amount of turning or change in direction.

So learning, using, or applying those ideas or tools in math, we get to solve problems. And this place can help. It can help you build and run your own math engine in your brain, and thus, can help achieve your solid math skill.

It is however, not a magic place giving you a math skill of high caliber overnight. And it’s just a place of examples, and can have many mistakes. I’m a human, too. There’s no mojo making a math genius, and math is science, and is full of facts and ideas. And it is after all, not me, not your teacher, and not any book but you who actually make you put together some of those facts and ideas, and understand it.

Putting facts and ideas together, understanding it, and taking care of what you have learned doing examples, you grow your math. And this place is ready to help and can help you help yourself.

Solving a problem, we put it many different ways. For instance, while expanding or reducing it, or modifying or converting it, we keep searching for the solution, keep approaching the solution, and eventually, can get there.

It is often the case a problem itself is the solution. We can put a problem in many different ways, and eventually, can end up with the solution. How come then is the solution no other than the problem?

For instance, the solution to 3232 ÷ 101 is 32. And we can put it this way:

$3232 \div 101 = \frac{3232}{101} = \frac{32 \text{ x } 101}{101} = \frac{32}{1} = 32 \Rightarrow 3232 \div 101 = 32.$

And we can get this, too: 32 = 3232 ÷ 101.   How?

$32 = \frac{32}{1} = \frac{32 \text{ x } 101}{101} = \frac{3232}{101} =$ 3232/101 = 3232 ÷ 101.

Too easy?

For another instance, the solution to $\boldsymbol{ax}^2 + \boldsymbol{bx} + \boldsymbol{c} = \mathbf{0}$ is

$\boldsymbol{x} = \frac{-\boldsymbol{b} \pm \sqrt{\boldsymbol{b}^2 - 4\boldsymbol{ac}}}{2\boldsymbol{a}}$, which is called the quadratic formula.

How come then is the solution no other than the problem?

We can put it this way:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow 2ax = -b \pm \sqrt{b^2 - 4ac}$$

$$\Rightarrow 2ax + b = \pm\sqrt{b^2 - 4ac} \Rightarrow (2ax + b)^2 = b^2 - 4ac$$

$$\Rightarrow 4a^2x^2 + 4abx + b^2 = b^2 - 4ac \Rightarrow 4a^2x^2 + 4abx = -4ac$$

$$\Rightarrow ax^2 + bx = -c \Rightarrow ax^2 + bx + c = 0.$$

And we can get this, too:

$$ax^2 + bx + c = 0 \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{How?}$$

$$ax^2 + bx + c = a(x^2 + \frac{b}{a}x) + c = a(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}) + c$$

$$= a(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}) - \frac{b^2}{4a} + c = a(x + \frac{b}{2a})^2 - \frac{b^2 - 4ac}{4a} = 0$$

$$\Rightarrow a(x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a} \Rightarrow (x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2}$$

$$\Rightarrow x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} \Rightarrow x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So when solving a problem, we expand or reduce it, modify or convert it, and keep searching for and approaching the solution. Then eventually, we can get there. What then do we do when doing the set of processes above?

We do algebra. You need a strong foundation in algebra. So if a problem is well defined, that is, if it makes sense, we should be able to get it solved the way as follows.

A problem $\Rightarrow$ **…** $\Rightarrow$ **…** $\Rightarrow$ **the solution**,

and thus, **the problem** $\Rightarrow$ **the solution**.

So solving a problem, we put it many different ways so that we can get to the solution. And that's the way math runs.

Many more examples are available for free, and you can get them downloaded at **www.runmath.com**. So visit the website, get them downloaded, and do the examples. The website is getting updated weekly or monthly.

May your math run very well.

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